N6.1

Запишем уравнения:

$$\frac{\partial\rho}{\partial t} + \operatorname{div}\bar{j} = 0 \ ; \quad \bar{j} = \sigma\bar{E} \ ; \quad \bar{D} = \varepsilon E \ , \quad \operatorname{div}\bar{D} = 4\pi\rho \ \Rightarrow$$

$$\bar{j} = \frac{\sigma}{\varepsilon}\bar{D} \ ; \quad \operatorname{div}\bar{j} = \frac{\sigma}{\varepsilon}\operatorname{div}\bar{D} = \frac{4\pi\rho\sigma}{\varepsilon}$$

$$\frac{\partial\rho}{\partial t} + \frac{4\pi\sigma}{\varepsilon}\rho = 0 \Rightarrow \frac{\partial\rho}{\rho} = -\frac{4\pi\sigma}{\varepsilon}\partial t \Rightarrow \rho = C\exp\left\{-\frac{4\pi\sigma}{\varepsilon}t\right\}$$

Из начальных условий: $\rho(\bar{r}, t=0) = \rho_0(\bar{r}) \Rightarrow$

$$\rho(\bar{r},t) = \rho_0(\bar{r})\exp\left\{-\frac{4\pi\sigma}{\varepsilon}t\right\}$$

Найдем изменения поле E:

$$\rho = \frac{\operatorname{div}\bar{D}}{4\pi} = \frac{\varepsilon\operatorname{div}\bar{E}}{4\pi}$$

$$\frac{\partial}{\partial t}\frac{\varepsilon\operatorname{div}\bar{E}}{4\pi} + \operatorname{div}\bar{j} = \frac{\partial}{\partial t}\frac{\varepsilon\operatorname{div}\bar{E}}{4\pi} + \sigma\operatorname{div}E = \operatorname{div}\left(\frac{\varepsilon}{4\pi}\frac{\partial\bar{E}}{\partial t} + \sigma\bar{E}\right) = 0 \Rightarrow$$

$$\Rightarrow \frac{\varepsilon}{4\pi}\frac{\partial\bar{E}}{\partial t} + \sigma\bar{E} = 0 \ ; \quad E(\bar{r},t) = \bar{E}_0(\bar{r})\exp\left\{-\frac{4\pi\sigma}{\varepsilon}t\right\}$$

$$\operatorname{rot}\bar{H} = \frac{4\pi}{c}\bar{j} + \frac{1}{c}\frac{\partial\bar{D}}{\partial t} = \frac{1}{c}\left(4\pi\sigma\bar{E} + \varepsilon\frac{\partial\bar{E}}{\partial t}\right)$$

$$\varepsilon\frac{\partial\bar{E}}{\partial t} = -4\pi\sigma\bar{E} \Rightarrow \operatorname{rot}\bar{H} = 0 \Rightarrow |\bar{H}| = 0.$$

N 6.8

$$\operatorname{rot}\bar{H} = \frac{4\pi}{c}\bar{j} + \frac{1}{c}\frac{\partial\bar{D}}{\partial t} ; \ \cancel{\frac{4\pi}{c}}\sigma$$

Для комплексных амплитуд

$$\operatorname{rot} \bar H = \frac{4\pi}{c}\bar j + \frac{i\omega}{c}\bar D = \frac{4\pi}{c}\sigma \bar E + \frac{i\omega}{c}\varepsilon \bar E =$$

$$= \frac{i\omega}{c}\left(\frac{4\pi\sigma}{i\omega} + \varepsilon\right)\bar E = \frac{i\omega}{c}\left(\varepsilon - i\frac{4\pi\sigma}{\omega}\right)\bar E = \frac{i\omega}{c}\varepsilon_k \bar E$$

Учитывая, что $\sigma \gg \omega$

$$\varepsilon_k \approx -i\frac{4\pi\sigma}{\omega}$$

$$\begin{cases} \operatorname{rot}\bar H = \frac{4\pi}{c}\bar j + \frac{1}{c} i\omega \bar D \ \cancel{\ \ } = \frac{i\omega}{c}\varepsilon_k \bar E \\ \operatorname{rot}\bar E = -\frac{\mu}{c} i\omega \bar H \qquad | \operatorname{rot} \end{cases}$$

$\cancel{\operatorname{rot}\operatorname{rot}\bar H = \frac{4\pi}{c}\varepsilon_k \operatorname{rot}\bar E}$ $\qquad \operatorname{rot}\operatorname{rot}\bar E = -\mu\frac{i\omega}{c}\operatorname{rot}\bar H$

$\cancel{\operatorname{grad}\operatorname{div}\bar H -}$ $\qquad \operatorname{grad}\operatorname{div}\bar E - \Delta\bar E = -\mu\frac{i\omega}{c}\operatorname{rot}\bar H$

$$-\Delta\bar E = -\frac{i\omega}{c}\mu\frac{i\omega}{c}\varepsilon_k\bar E, \quad \Delta\bar E + \frac{\omega^2}{c^2}\mu\varepsilon_k\bar E = 0$$

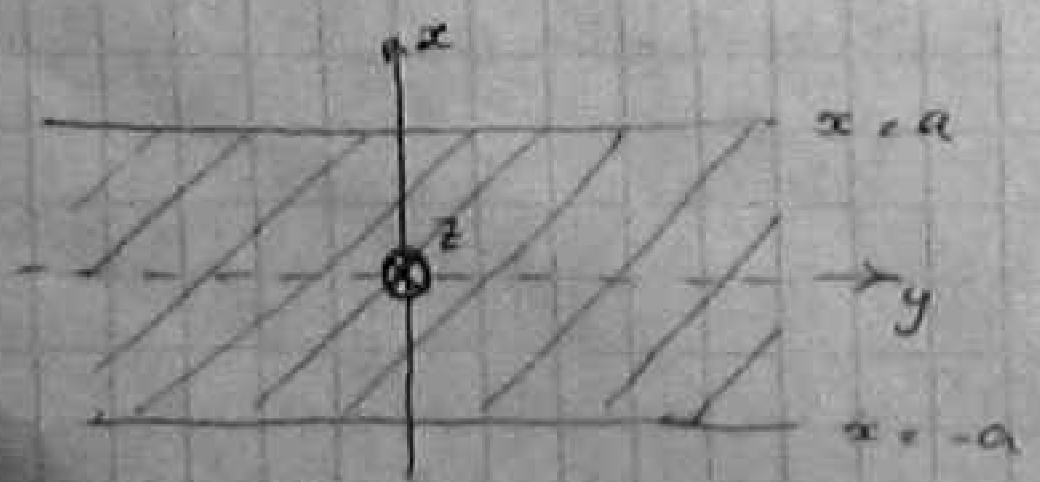
$$\bar E = \bar E(x)$$

$$\frac{d^2E}{dx^2} + \frac{\omega^2}{c^2}\mu\varepsilon_k\bar E = 0$$

$$\operatorname{div}\bar D = 0 \Rightarrow \operatorname{div}\bar E = 0 \Rightarrow \frac{dE_x}{dx} = 0 \Rightarrow E_x = \text{const}$$

$$\frac{d^2E_x}{dx^2} + \frac{\omega^2}{c^2}\mu\varepsilon_k E_x = 0 \Rightarrow E_x = 0$$

Тангенсальная составляющая задана как $E_y \Rightarrow E_z = 0$



$$\frac{d^2E_y}{dx^2} + \frac{\omega^2}{c^2}\mu\varepsilon_k E_y = 0$$

$$E_y = C_1\exp\left\{\frac{\omega}{c}\sqrt{\mu\varepsilon_k}\,x\right\} + C_2\exp\left\{-\frac{\omega}{c}\sqrt{\mu\varepsilon_k}\,x\right\}$$

$$\sqrt{\mu\varepsilon_k} = \sqrt{\mu\left(-i\frac{4\pi\sigma}{\omega}\right)} = \pm\sqrt{\frac{2\pi\sigma\mu}{\omega}}(1+i)$$

$$E_y = C_1\,\mathrm{sh}\left\{\frac{\omega}{c}\sqrt{\frac{2\pi\sigma\mu}{\omega}}(1+i)x\right\} + C_2\,\mathrm{ch}\left\{+\frac{\omega}{c}\sqrt{\frac{2\pi\sigma\mu}{\omega}}(1+i)x\right\}$$

$E_y(a) = E_y(-a) = E_0$ – четная функция

$$E_y = C_2\,\mathrm{ch}\left\{\frac{\omega}{c}\sqrt{\frac{2\pi\sigma\mu}{\omega}}(1+i)x\right\} = C_2\,\mathrm{ch}\left\{\frac{1}{c}\sqrt{2\pi\sigma\mu\omega}(1+i)x\right\} =$$

$$= C_2\,\mathrm{ch}\left\{\frac{x}{\delta}(1+i)\right\}$$

$$E_y(a) = C_2\,\mathrm{ch}\left\{\frac{a}{\delta}(1+i)\right\} = E_0, \quad C_2 = E_0\,\mathrm{ch}^{-1}\left\{\frac{a}{\delta}(1+i)\right\}$$

$$E_y = E_0\,\mathrm{ch}^{-1}\left\{\frac{a}{\delta}(1+i)\right\}\mathrm{ch}\left\{\frac{x}{\delta}(1+i)\right\} =$$

$$\cancel{\ \ } = E_0\frac{e^{\frac{x}{\delta}}e^{\frac{ix}{\delta}} + e^{-\frac{x}{\delta}}e^{-\frac{ix}{\delta}}}{e^{\frac{a}{\delta}}e^{\frac{ia}{\delta}} + e^{-\frac{a}{\delta}}e^{-\frac{ia}{\delta}}} =$$

$$= E_0\frac{e^{\frac{x}{\delta}}\left(\cos\frac{x}{\delta} + i\sin\frac{x}{\delta}\right) + e^{-\frac{x}{\delta}}\left(\cos\frac{x}{\delta} - i\sin\frac{x}{\delta}\right)}{e^{\frac{a}{\delta}}\left(\cos\frac{a}{\delta} + i\sin\frac{a}{\delta}\right) + e^{-\frac{a}{\delta}}\left(\cos\frac{a}{\delta} - i\sin\frac{a}{\delta}\right)} =$$

$$= E_0\frac{\mathrm{ch}\frac{x}{\delta}\cos\frac{x}{\delta} + i\,\mathrm{sh}\frac{x}{\delta}\sin\frac{x}{\delta}}{\mathrm{ch}\frac{a}{\delta}\cos\frac{a}{\delta} + i\,\mathrm{sh}\frac{a}{\delta}\sin\frac{a}{\delta}};$$

$$= \cancel{E_0\,\mathrm{ch}^2\frac{x}{\delta}\cos} \quad \cancel{E_0\frac{\mathrm{ch}^2\frac{x}{\delta}\cos^2\frac{x}{\delta} - \mathrm{sh}^2\frac{x}{\delta}\sin^2\frac{x}{\delta}}{\mathrm{ch}^2\frac{a}{\delta}\cos^2\frac{a}{\delta} + \mathrm{sh}^2\frac{a}{\delta}\sin^2\frac{a}{\delta}}}$$

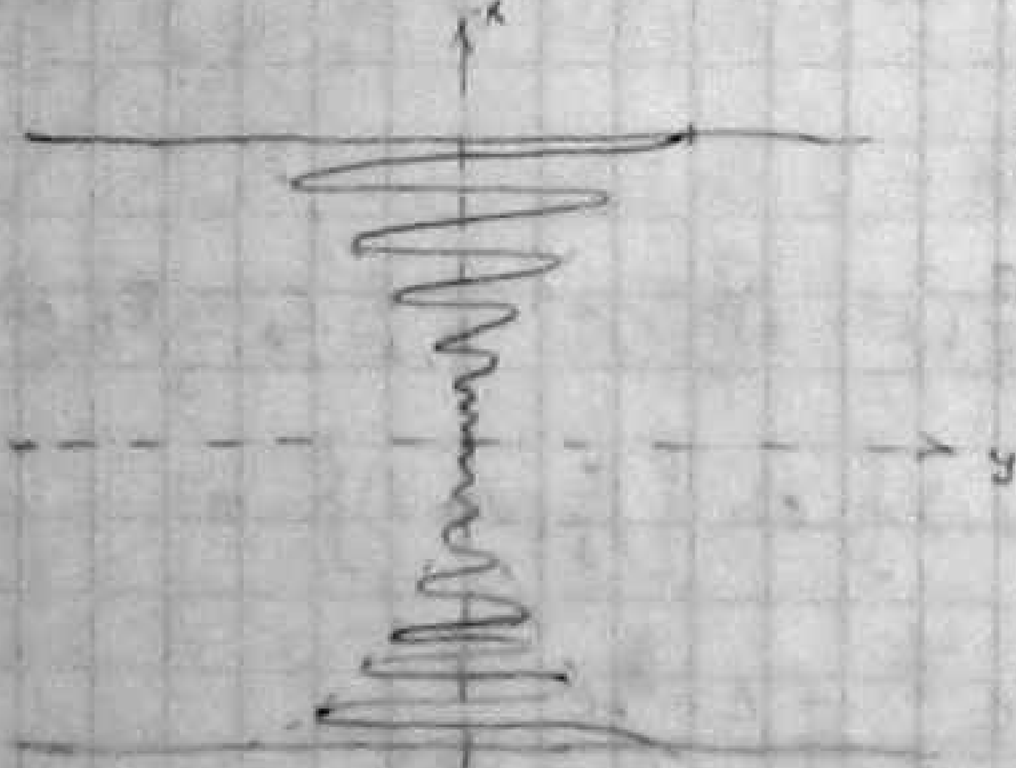
а) $a \gg \delta \Rightarrow \frac{a}{\delta} \gg 1$

а) $a \gg \delta$, $\frac{a}{\delta} \gg 1$

$$E_y = E_0 \frac{e^{\frac{x}{\delta}} e^{\frac{ix}{\delta}} + e^{-\frac{x}{\delta}} e^{-\frac{ix}{\delta}}}{e^{\frac{a}{\delta}} e^{\frac{ia}{\delta}}};$$

$$E_y e^{i\omega t} = E_0 e^{\frac{x}{\delta}-\frac{a}{\delta}} e^{i\left(\frac{x}{\delta}-\frac{a}{\delta}+\omega t\right)} + E_0 e^{-\frac{x}{\delta}-\frac{a}{\delta}} e^{i\left(-\frac{x}{\delta}-\frac{a}{\delta}+\omega t\right)} =$$

$$\mathrm{Re}\{E_y e^{i\omega t}\} = E_0 e^{\frac{x}{\delta}-\frac{a}{\delta}} \cos\left(\omega t + \frac{x}{\delta} - \frac{a}{\delta}\right) + \underbrace{E_0 e^{-\frac{x}{\delta}-\frac{a}{\delta}} \cos\left(\omega t - \frac{x}{\delta} - \frac{a}{\delta}\right)}_{\approx 0}$$



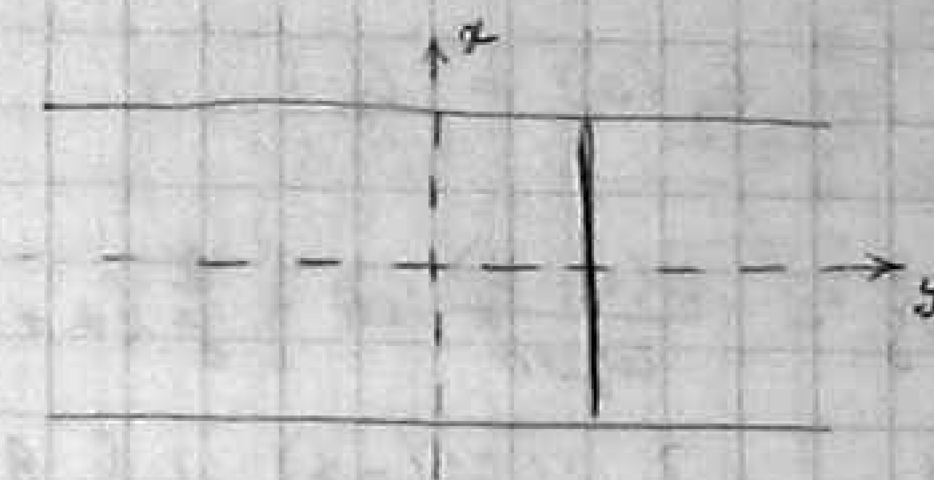
б) $a \ll \delta$, $\frac{a}{\delta} \ll 1$

$$E_y \approx E_0 \frac{\left(1+\frac{x}{\delta}\right) e^{\frac{ix}{\delta}} + \left(1-\frac{x}{\delta}\right) e^{-\frac{ix}{\delta}}}{\left(1+\frac{a}{\delta}\right) e^{\frac{ia}{\delta}} + \left(1-\frac{a}{\delta}\right) e^{-\frac{ia}{\delta}}} =$$

$$= E_0 \frac{\left(1+\frac{x}{\delta}\right)\left(1+\frac{ix}{\delta}\right) + \left(1-\frac{x}{\delta}\right)\left(1-\frac{ix}{\delta}\right)}{\left(1+\frac{a}{\delta}\right)\left(1+\frac{ia}{\delta}\right) + \left(1-\frac{a}{\delta}\right)\left(1-\frac{ia}{\delta}\right)} =$$

$$\approx E_0 \frac{1+\frac{x}{\delta}+\frac{ix}{\delta}+i\frac{x^2}{\delta^2}+1-\frac{x}{\delta}-\frac{ix}{\delta}+i\frac{x^2}{\delta^2}}{1+\frac{a}{\delta}+\frac{ia}{\delta}+i\frac{a^2}{\delta^2}+1-\frac{a}{\delta}-\frac{ia}{\delta}+i\frac{a^2}{\delta^2}} = E_0 \frac{2+i\frac{x^2}{\delta^2}}{2+i\frac{a^2}{\delta^2}} \approx E_0$$

$$\mathrm{Re}\{E_y e^{i\omega t}\} = E_0 \frac{\cancel{2\cos\omega t - \frac{x^2}{\delta^2}\sin\omega t}}{\cancel{2\cos\omega t - \frac{a^2}{\delta^2}}} = E_0 \cos\omega t$$



№6.10

Из соображений предыдущей задачи $E_x = E_z = 0$

а) $E_y(-a) = E_0$

$E_y(0) = 0$

$$E_y = C_1 \,\mathrm{sh}\left\{\frac{x}{\delta}(1+i)\right\} + C_2 \,\mathrm{ch}\left\{\frac{x}{\delta}(1+i)\right\}$$

$E_y(x=0) = C_2 = 0;$

$E_y(-a) = -C_1 \,\mathrm{sh}\left\{\frac{a}{\delta}(1+i)\right\} = E_0 \Rightarrow$

$C_1 = -E_0 \,\mathrm{sh}^{-1}\left\{\frac{a}{\delta}(1+i)\right\}$

$$E_y = -E_0 \frac{\mathrm{sh}\left\{\frac{x}{\delta}(1+i)\right\}}{\mathrm{sh}\left\{\frac{a}{\delta}(1+i)\right\}}$$

б) $E_y(-a) = E_0$

$$E_y = C_1 \,\mathrm{sh}\left\{\frac{x}{\delta}(1+i)\right\} + C_2 \,\mathrm{ch}\left\{\frac{x}{\delta}(1+i)\right\}$$

$$\mathrm{rot}\,\bar{E} = -\frac{i\omega}{c}\mu \bar{H}$$

$$\mathrm{rot}\,\bar{E} = \begin{vmatrix} \bar{x}_0 & \bar{y}_0 & \bar{z}_0 \\ \frac{\partial}{\partial x} & 0 & 0 \\ 0 & E_y & 0 \end{vmatrix} = \bar{z}_0 \frac{\partial E_y}{\partial x}, \quad \bar{H} = \frac{ci}{\omega\mu} \bar{z}_0 \frac{\partial E_y}{\partial x}$$

$$H_z = \frac{ci}{\omega\mu}\,\frac{i+1}{\delta}\left[C_1 \operatorname{ch}\left\{\frac{x}{\delta}(1+i)\right\} + C_2 \operatorname{sh}\left\{\frac{x}{\delta}(1+i)\right\}\right]$$

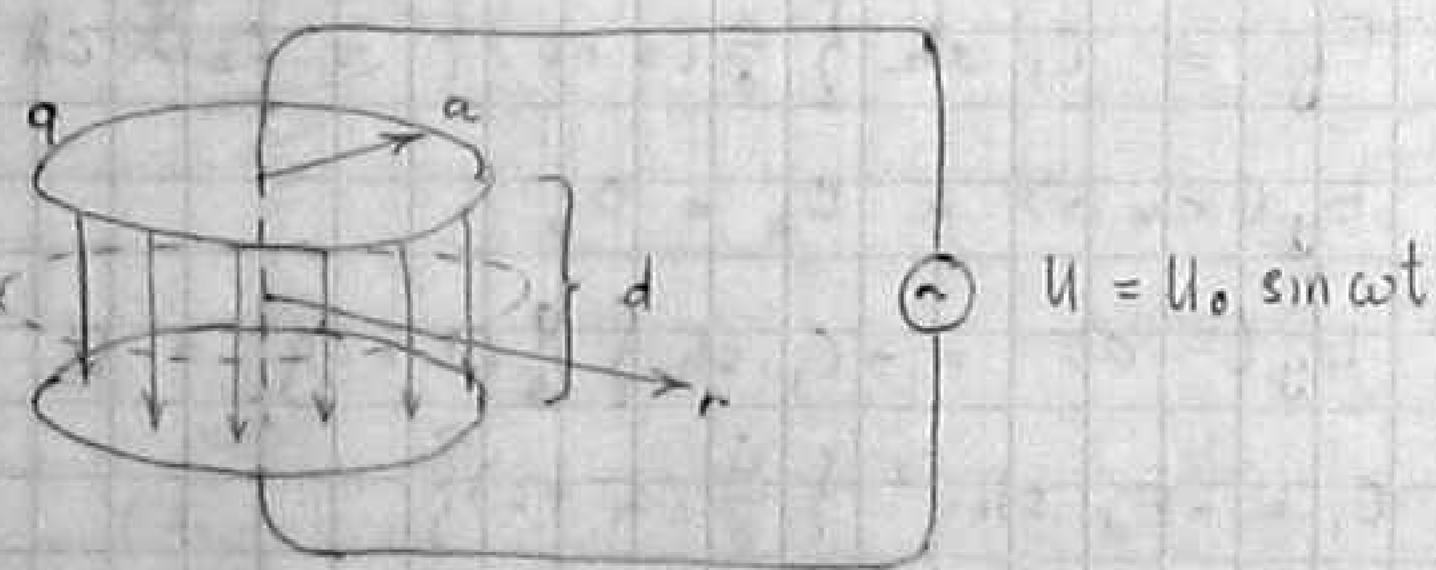
$$H_z(0) = \frac{c(i-1)}{\omega\mu\delta}\,C_1 = E_y(0) = C_2$$

$$E_y = C_1\left[\operatorname{sh}\left\{\frac{x}{\delta}(1+i)\right\} + \frac{c(i-1)}{\omega\mu\delta}\operatorname{ch}\left\{\frac{x}{\delta}(1+i)\right\}\right]$$

$$E_y(-a) = C_1\left[-\operatorname{sh}\left\{\frac{a}{\delta}(1+i)\right\} + \frac{c(i-1)}{\omega\mu\delta}\operatorname{ch}\left\{\frac{a}{\delta}(1+i)\right\}\right] = E_0$$

$$E_y = E_0 \cdot \frac{\frac{c(i-1)}{\omega\mu\delta}\operatorname{ch}\left\{\frac{x}{\delta}(1+i)\right\} + \operatorname{sh}\left\{\frac{x}{\delta}(1+i)\right\}}{\frac{c(i-1)}{\omega\mu\delta}\operatorname{ch}\left\{\frac{a}{\delta}(1+i)\right\} - \operatorname{sh}\left\{\frac{a}{\delta}(1+i)\right\}}$$

N6.12



$$q = CU = CU_0 \sin\omega t$$

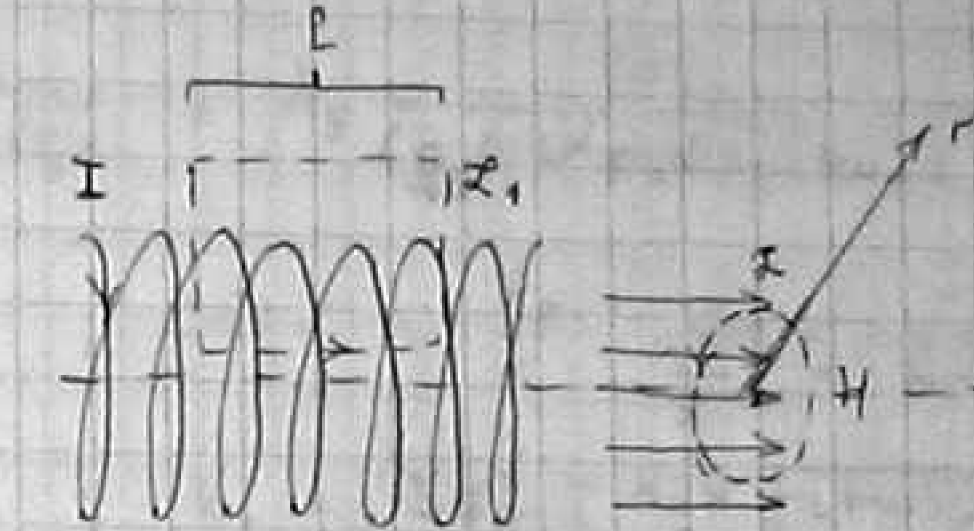
$$Ed = U_0 \sin\omega t\,, \quad E = \frac{U_0 \sin\omega t}{d}$$

$$\int_L \bar{H}\,d\bar{l} = \frac{1}{c}\frac{\partial}{\partial t}\int_S \bar{D}\,d\bar{S} = \frac{\varepsilon}{c}\frac{\partial}{\partial t}\int_S \bar{E}\,d\bar{S} = \frac{\varepsilon}{c}\frac{\partial}{\partial t}(ES) =$$

$$= \frac{\varepsilon}{c}\frac{\partial}{\partial t}\left(\frac{U_0 \sin\omega t}{d}\,\pi r^2\right) = \frac{\omega\varepsilon U_0 \pi r^2 \cos\omega t}{cd}$$

$$\int_L \bar{H}\,d\bar{l} = 2\pi r H\,, \quad H = \frac{\omega\varepsilon U_0 r \cos\omega t}{2cd}$$

N6.13



$$\oint_{L_1} \bar{H}\,d\bar{l} = \frac{4\pi}{c} I_\Sigma$$

$$\oint_{L_1} \bar{H}\,d\bar{l} = lH\,; \quad I_\Sigma = nl I_0 \sin\omega t\,; \quad H = \frac{4\pi n I_0}{c}\sin\omega t$$

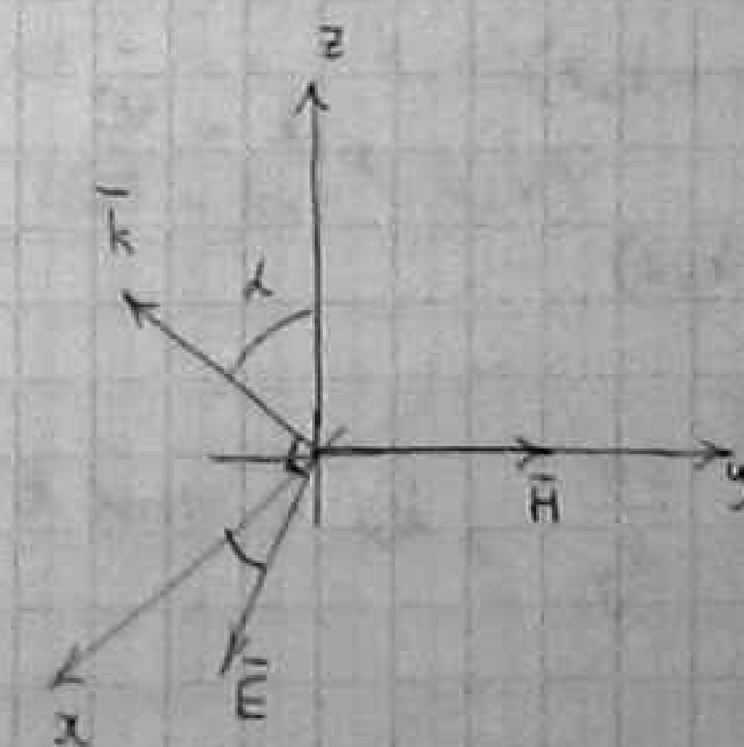
$$\oint_L \bar{E}\,d\bar{l} = -\frac{1}{c}\frac{\partial}{\partial t}\int_S \bar{B}\,d\bar{S}$$

$$\oint_L \bar{E}\,d\bar{l} = 2\pi r E\,; \quad -\frac{1}{c}\frac{\partial}{\partial t}\int_S \bar{B}\,d\bar{S} = -\frac{\mu}{c}\frac{\partial}{\partial t}(H\pi r^2) =$$

$$= -\frac{\mu}{c}\,\frac{4\pi}{c}\,n I_0\,\pi r^2\,\omega\cos\omega t$$

$$E = -\frac{4\pi\mu n I_0 r\omega}{2c^2}\cos\omega t = -\frac{2\pi\mu n I_0 r\omega}{c^2}\cos\omega t$$

N7.1



а) $\bar{E} = \bar{E}_0 e^{i(\omega t - \bar{k} r)}$

$E_x = \cos\alpha\, |\bar{E}_0|\, e^{i(\omega t - k_x x - k_z z)}$

$\cos\alpha = \dfrac{k_z}{k}$; $\sin\alpha = \dfrac{k_x}{k}$

$E_x = \dfrac{k_z}{k} |\bar{E}_0|\, e^{i(\omega t - k_x x - k_z z)}$

$E_z = -\dfrac{k_x}{k} |\bar{E}_0|\, e^{i(\omega t - k_x x - k_z z)}$

$E_y = 0$

$H_x = 0$; $H_z = 0$

$\text{rot}\, \bar{E} = -\dfrac{i\omega}{c} \cancel{\mu} \bar{H}$; $\bar{H} = \dfrac{ic}{\omega} \text{rot}\, \bar{E}$

$$\text{rot}\, E = \begin{vmatrix} \bar{x}_0 & \bar{y}_0 & \bar{z}_0 \\ \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ E_x & 0 & E_z \end{vmatrix} = \bar{y}_0 \left(\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) =$$

$$= \bar{y}_0 \left(\frac{-ik_z^2}{k} |\bar{E}_0| e^{i(\omega t - k_x x - k_z z)} + \frac{-ik_x^2}{k} |\bar{E}_0| e^{i(\omega t - k_x x - k_z z)} \right) =$$

$$= \bar{y}_0 (-ik) |\bar{E}_0| e^{i(\omega t - k_x x - k_z z)}$$

$H_y = \dfrac{c}{\omega} k |\bar{E}_0| e^{i(\omega t - k_x x - k_z z)}$; $k = \dfrac{\omega}{c}$

$H_y = |\bar{E}_0| e^{i(\omega t - k_x x - k_z z)}$

б) $\lambda_x = \dfrac{2\pi}{k_x}$; $\lambda_y = \dfrac{2\pi}{k_y}$; $k_x = \sin\alpha\, k$; $k_z = \cos\alpha\, k$

$k = \dfrac{\omega}{c}\sqrt{\varepsilon\mu}$

$\lambda_x = \dfrac{2\pi c}{\sin\alpha\, \omega \sqrt{\varepsilon\mu}}$; $\lambda_y = \dfrac{2\pi c}{\cos\alpha\, \omega\sqrt{\varepsilon\mu}}$

в) $\lambda_x = \dfrac{2\pi}{k_x}$; $\cos\alpha = \dfrac{k_z}{k}$; $\sin\alpha = \sqrt{1 - \cos^2\alpha} = \sqrt{1 - \dfrac{k_z^2}{k^2}}$

$k = \dfrac{\omega}{c}\sqrt{\varepsilon\mu}$; $k_x = k\sin\alpha = \sqrt{k^2 - k_z^2}$; $\lambda_z = \dfrac{2\pi}{k_z}$

$k_z = \dfrac{2\pi}{\lambda_z}$; $k_x = \sqrt{\dfrac{\omega^2}{c^2}\varepsilon\mu - \dfrac{4\pi^2}{\lambda_z^2}}$

$$\lambda_x = \frac{2\pi}{\sqrt{\frac{\omega^2}{c^2}\varepsilon\mu - \frac{4\pi^2}{\lambda_z^2}}} = \frac{1}{\sqrt{\frac{\omega^2 \varepsilon\mu}{4\pi^2 c^2} - \frac{1}{\lambda_z^2}}} = \frac{\lambda_z}{\sqrt{\frac{\lambda_z^2 \omega^2 \varepsilon\mu}{4\pi^2 c^2} - 1}}$$

г) $v^{(z)} = v\cos\alpha$; $v = \dfrac{c}{\sqrt{\varepsilon\mu}}$; $\lambda_x = \dfrac{\lambda}{\sin\alpha}$; $\lambda = \dfrac{c}{\sqrt{\varepsilon\mu}} \dfrac{2\pi}{\omega}$

$\lambda_x = \dfrac{c}{\sqrt{\varepsilon\mu}} \cdot \dfrac{2\pi}{\omega} \cdot \dfrac{1}{\sin\alpha}$; $\cos\alpha = \dfrac{v^{(z)}}{v} = v^{(z)} \dfrac{\sqrt{\varepsilon\mu}}{c}$

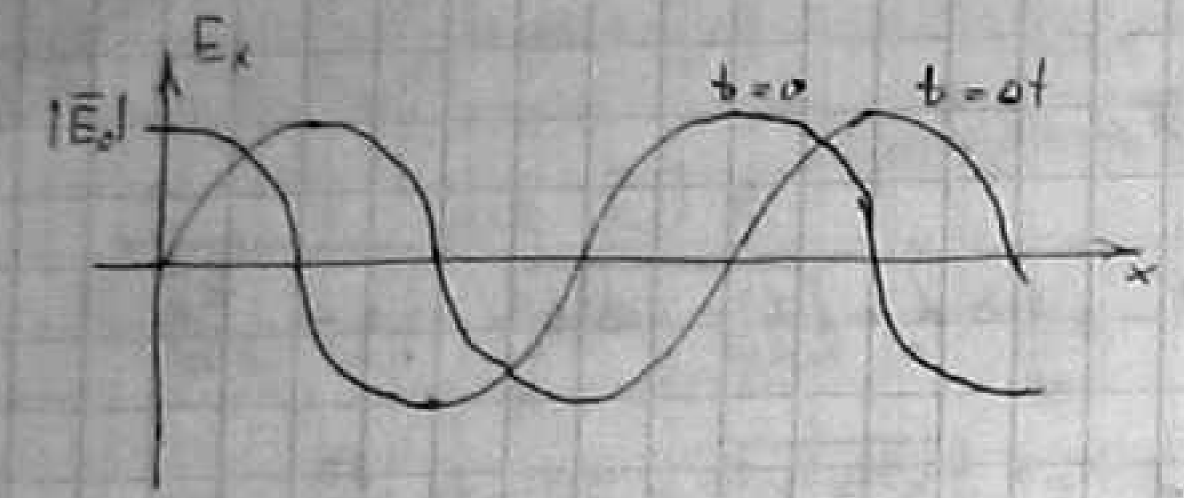
$\sin\alpha = \sqrt{1 - \dfrac{v^{(z)2}\varepsilon\mu}{c^2}}$

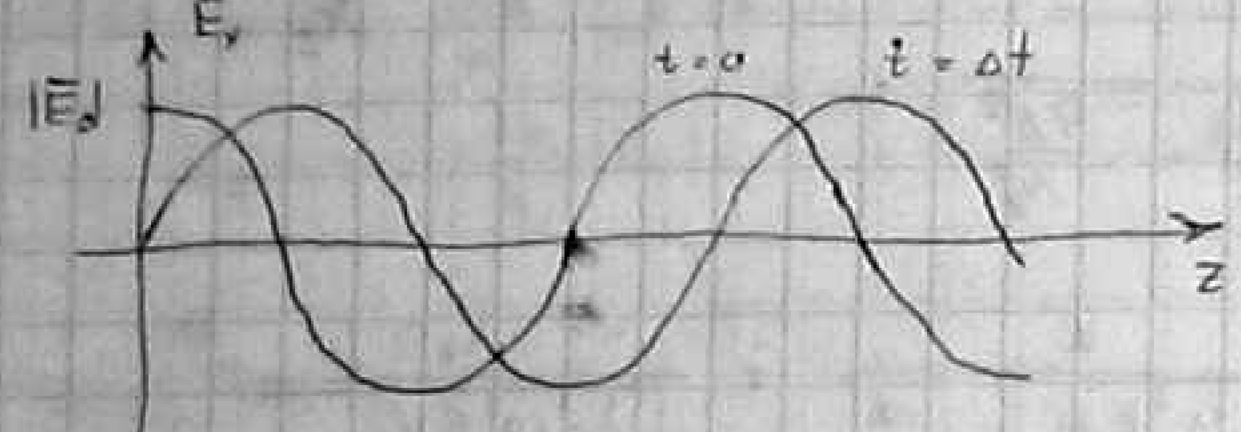
$$\omega = \frac{2\pi c}{\lambda_x \sqrt{\varepsilon\mu}\, \sin\alpha} = \frac{2\pi c}{\lambda_x \sqrt{\varepsilon\mu}} \left[1 - \frac{v^{(z)2}\varepsilon\mu}{c^2} \right]^{-1/2}$$

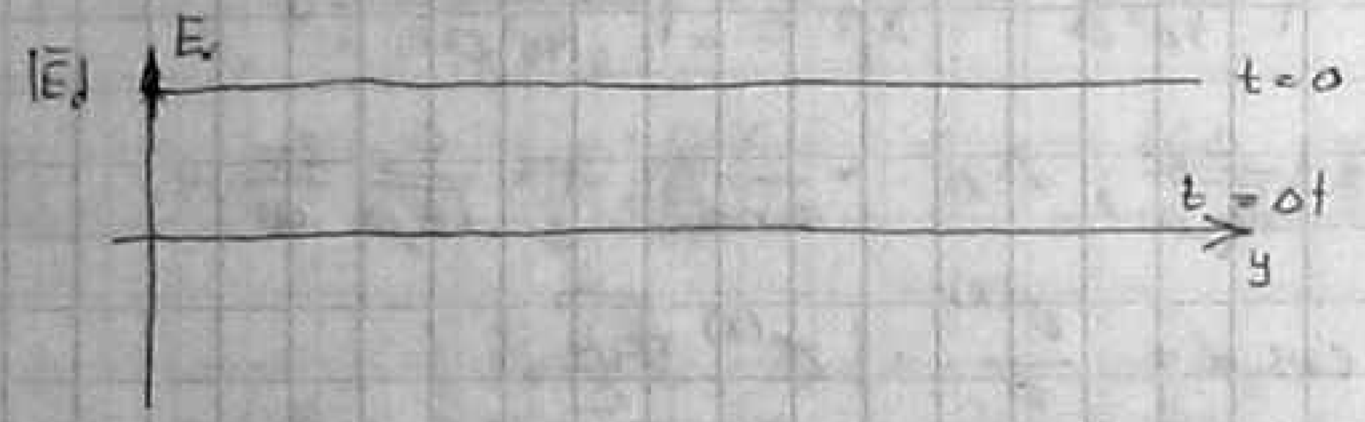
д) В случае $\bar{k} \parallel \bar{z}_0$; $k_z = k$; $k_x = 0$

$E_x = |\bar{E}_0| e^{i(\omega t - k_x x - k_z z)}$

$\# \ H_y = |\bar{E}_0| e^{i(\omega t - k_x x - k_z z)}$

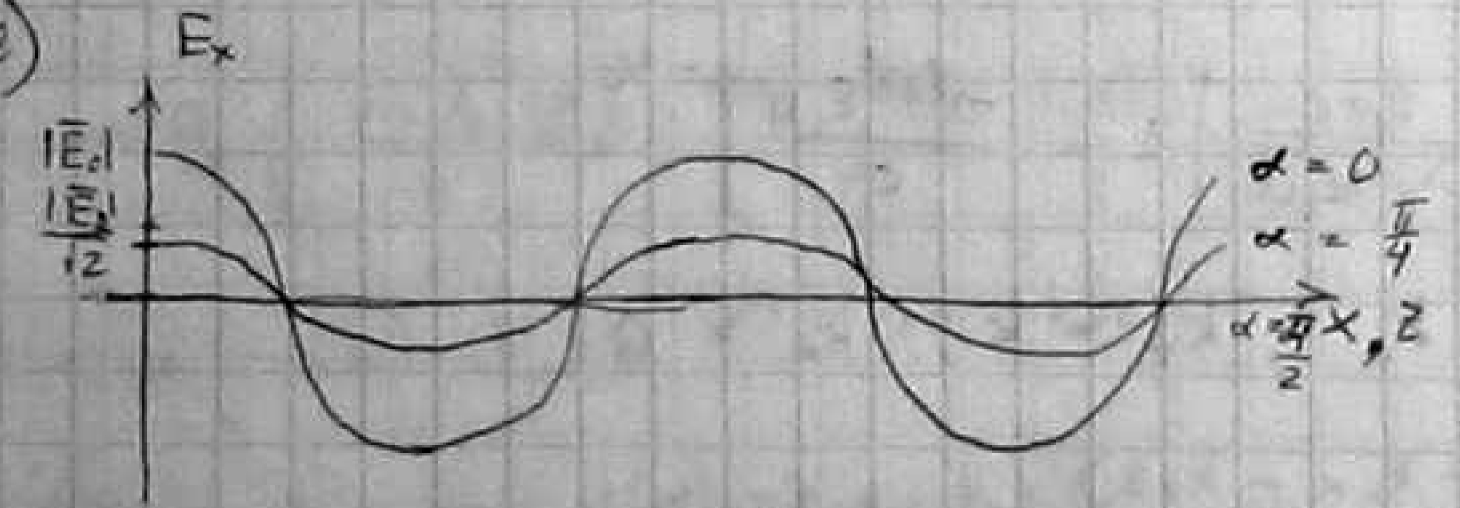


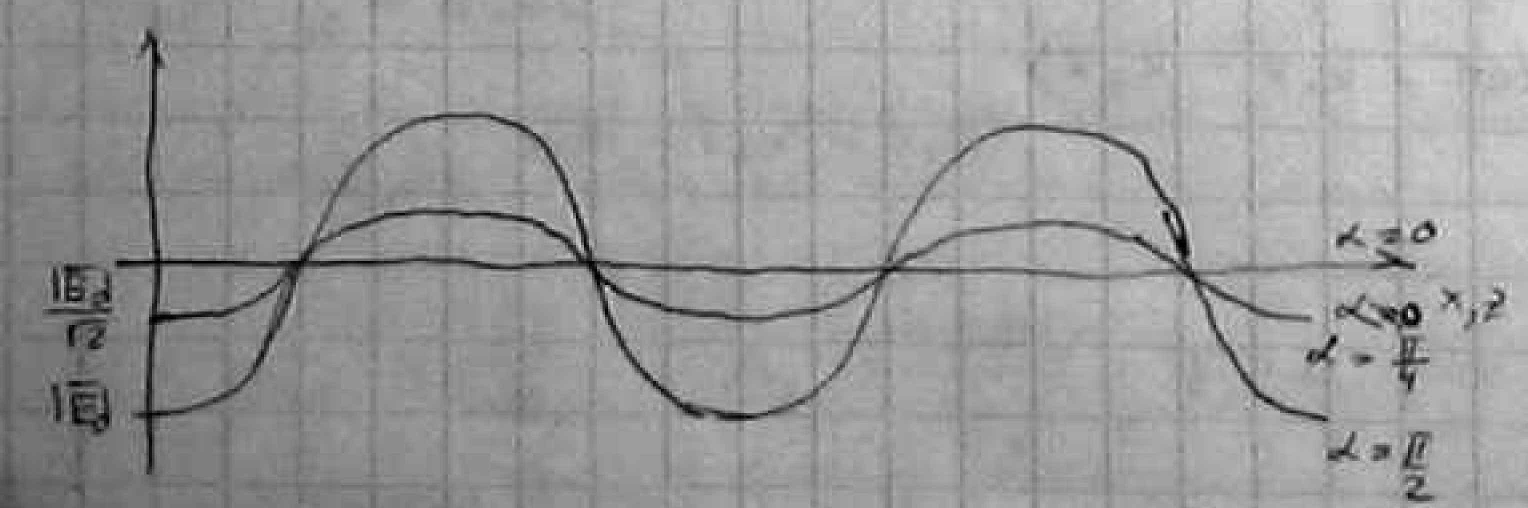




Аналогично и для H_y

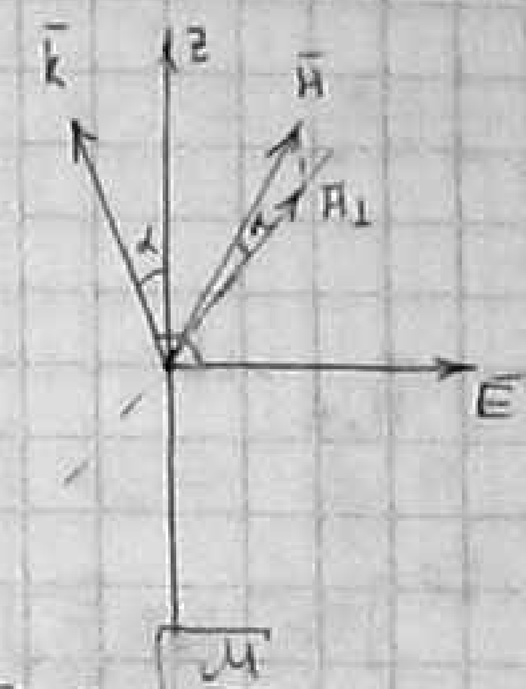
в)





№7.2

а) TE

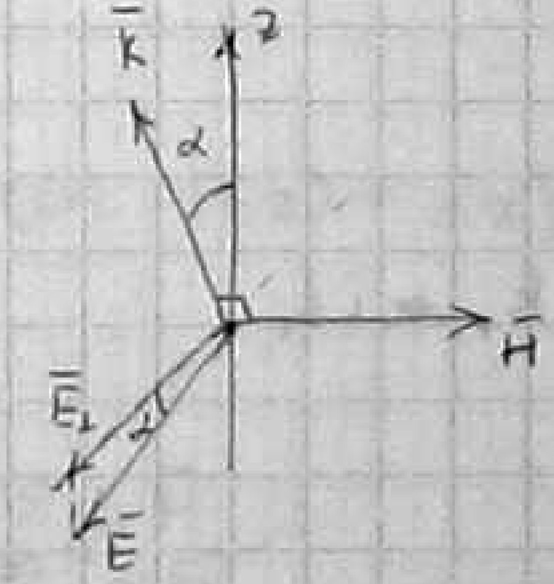


$E = \eta H$, где $\eta = \sqrt{\frac{\mu}{\varepsilon}}$

$E_\perp = E$, $H_\perp = H\cos\alpha$; $\eta \frac{H_\perp}{\cos\alpha} = E_\perp$,

$\eta_\perp = \frac{\eta}{\cos\alpha} = \sqrt{\frac{\mu}{\varepsilon}} \frac{1}{\cos\alpha}$

б) TM



$E_\perp = E\cos\alpha$; $H_\perp = H$, $\eta H_\perp = \frac{E_\perp}{\cos\alpha}$, $E_\perp = \eta\cos\alpha\, H_\perp$

$\eta_\perp = \sqrt{\frac{\mu}{\varepsilon}}\cos\alpha$

в) TEM

$E_\perp = E$, $H_\perp = H$; $E_\perp = \eta H_\perp$, $\eta_\perp = \sqrt{\frac{\mu}{\varepsilon}}$

№ 7.3

$$\overset{\text{ш}}{S} = \mathrm{Re}\,\frac{c}{8\pi}\left[\bar{E}\times\bar{H}^{*}\right]$$

$$\bar{E} = \bar{E}_0 e^{-i\bar{k}\bar{r}}$$

$$\bar{H} = \frac{\bar{E}}{\eta} = \sqrt{\frac{\varepsilon}{\mu}}\,\bar{E}_0 e^{-i\bar{k}\bar{r}} = \bar{H}_0 e^{-i\bar{k}\bar{r}}$$

$$S = \mathrm{Re}\,\frac{c}{8\pi}\left[\bar{E}_0 e^{-i\bar{k}\bar{r}}\times\bar{H}_0 e^{i\bar{k}\bar{r}}\right] = \mathrm{Re}\,\frac{c}{8\pi}|\bar{E}_0|^2\sqrt{\frac{\varepsilon}{\mu}} =$$

$$= \frac{c}{8\pi}|\bar{E}_0|^2\sqrt{\frac{\varepsilon}{\mu}};\quad |\bar{E}_0| = \left(\frac{8\pi}{c}\sqrt{\frac{\mu}{\varepsilon}}\right)^{1/2}$$

$$|\bar{H}_0| = \sqrt{\frac{\varepsilon}{\mu}}\left(\frac{8\pi}{c}\sqrt{\frac{\mu}{\varepsilon}}\right)^{1/2} = \left(\frac{8\pi}{c}\sqrt{\frac{\varepsilon}{\mu}}\right)^{1/2}$$

№ 7.5

a) $$\bar{E} = \bar{E}_0 e^{-i\bar{k}\bar{r}}$$

$$k = \frac{\omega}{c}\sqrt{\varepsilon_\mu\mu} = \frac{\omega}{c}\sqrt{\varepsilon} = k_r - ik_i$$

$$\bar{E} = \bar{E}_0 e^{-\bar{k}_i\bar{r}} e^{-i\bar{k}_r\bar{r}}$$

Для скорости волнового фронта $v = \frac{\omega}{k_r}$; $k_r = \frac{\omega}{v}$

Для затухания $k_i L = 1$, $k_i = \frac{1}{L}$

$$\frac{\omega^2}{c^2}(\varepsilon_r + i\varepsilon_i) = k_r^2 - 2ik_rk_i - k_i^2$$

$$\frac{\omega^2}{c^2}\varepsilon_r = k_r^2 - k_i^2,\quad \varepsilon_r = \frac{c^2}{\omega^2}(k_r^2 - k_i^2)$$

$$\varepsilon_r = \frac{c^2}{\omega^2}\left(\frac{\omega^2}{v^2} - \frac{1}{L^2}\right) = \frac{c^2}{v^2} - \frac{c^2}{\omega^2L^2}$$

$$\frac{\omega^2}{c^2}\varepsilon_i = -2k_rk_i;\quad \frac{\omega^2}{c^2}\varepsilon_i = -2\frac{\omega}{v}\frac{1}{L};$$

$$\varepsilon_i = -2\frac{c^2}{\omega^2}\frac{\omega}{v}\frac{1}{L} = -\frac{2c^2}{\omega v L}$$

~~№ 7.7~~ б)

$$E_0 = \eta H_0 = \sqrt{\frac{\mu}{\varepsilon}}H_0 = \frac{1}{\sqrt{\varepsilon}}H_0$$

$$E_0 = pH_0e^{i\varphi}\quad \cancel{\frac{E_0}{p}e^{i\varphi}}$$

$$\frac{1}{\sqrt{\varepsilon}} = pe^{i\varphi};\quad \frac{1}{\varepsilon} = p^2e^{2i\varphi};\quad \varepsilon = \frac{1}{p^2}e^{-2i\varphi}$$

$$\varepsilon_r + i\varepsilon_i = \frac{1}{p^2}(\cos 2\varphi - i\sin 2\varphi)$$

$$\varepsilon_r = \frac{\cos 2\varphi}{p^2},\quad \varepsilon_i = -\frac{\sin 2\varphi}{p^2}$$

№ 7.7

$$E_y = E_0\exp\{i(\omega t - hz) - \varkappa x\}$$

$$E_x = E_z = 0$$

$$\mathrm{rot}\,\bar{E} = -\frac{i\omega}{c}\mu\bar{H}$$

$$\bar{H} = \frac{ic}{\mu\omega}\mathrm{rot}\,\bar{E}$$

$$\mathrm{rot}\,\bar{E} = \begin{vmatrix} \bar{x}_0 & \bar{y}_0 & \bar{z}_0 \\ \frac{\partial}{\partial x} & 0 & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix} =$$

$$= -\bar{x}_0\frac{\partial E_y}{\partial z} + \bar{z}_0\frac{\partial E_y}{\partial x}$$

$$H_x = -\frac{ic}{\mu\omega}(-ih)E_0\exp\{-ihz - \varkappa x\} =$$

$$= -\frac{ch}{\omega\mu} E_0 \exp\{-ihz - kx\} = \frac{ch}{\omega\mu} E_0 \exp\{-ihz - kx + i\pi\}$$

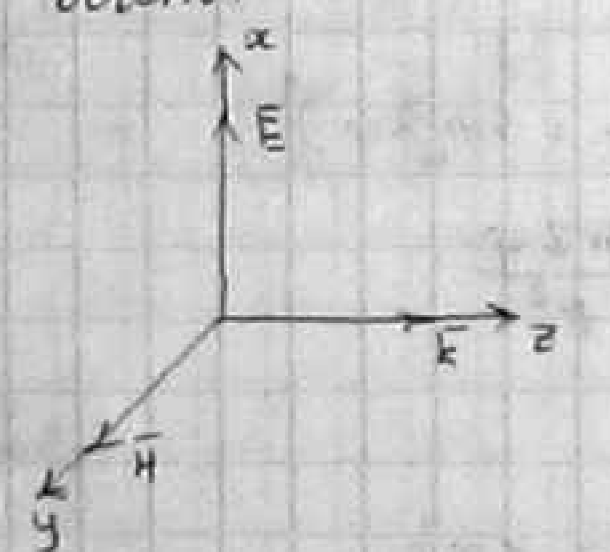
$$H_z = -\frac{ic}{\mu\omega}(-k)\exp\{-ihz - kx\} =$$

$$= \frac{ick}{\mu\omega}\exp\{-ihz - kx\} = \frac{ck}{\mu\omega}\exp\left\{-ihz - kx + i\frac{\pi}{2}\right\}$$

$\cancel{k_1 = \sqrt{\frac{\omega^2}{c^2}\varepsilon\mu}}$

№ 7.11

Для поперечной волны



$$\begin{cases} \operatorname{rot}\bar{E} = -\frac{1}{c} i\omega\mu \bar{H} & |\operatorname{rot} \\ \operatorname{rot}\bar{H} = \frac{1}{c} i\omega \bar{D} \end{cases}$$

$$-\Delta\bar{E} = -\frac{1}{c} i\omega\mu\left(\frac{1}{c} i\omega \bar{D}\right)$$

$$\Delta\bar{E} + \frac{\omega^2}{c^2}\mu\bar{D} = 0$$

$$\bar{D} = \varepsilon\bar{E} + \delta^2 \operatorname{grad}\operatorname{div}\bar{E}$$

$$\varepsilon = 1 - \frac{\omega_p^2}{\omega^2}; \quad \delta = \frac{V_T}{\omega}$$

$$\bar{E} = E_x \bar{x}_0 = \bar{x}_0 E_0 e^{-ikz}; \quad \operatorname{div}\bar{E} = 0$$

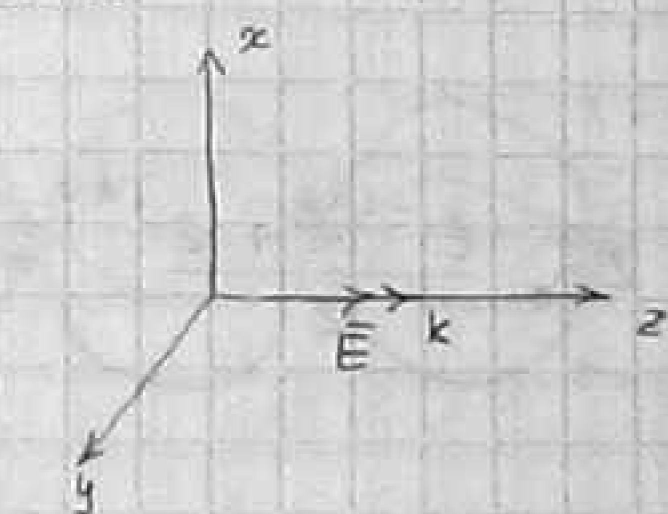
$$\bar{D} = \varepsilon \bar{x}_0 E_x$$

$$\Delta E_x + \frac{\omega^2}{c^2}\mu\varepsilon E_x = 0$$

$$k^2 = \frac{\omega^2}{c^2}\mu\varepsilon = \frac{\omega^2}{c^2}\mu\left(1 - \frac{\omega_p^2}{\omega^2}\right) = \frac{\omega^2\mu}{c^2} - \frac{\mu\omega_p^2}{c^2}$$

$$\omega^2 = \frac{k^2c^2}{\mu} + \omega_p^2$$

Для продольной волны



$$\begin{cases} \operatorname{rot}\bar{E} = -\frac{i\omega}{c}\mu\bar{H} & |\operatorname{rot} \\ \operatorname{rot}\bar{H} = \frac{i\omega}{c}\bar{D} = \frac{i\omega}{c}(\varepsilon\bar{E} + \delta^2 \operatorname{grad}\operatorname{div}\bar{E}) \end{cases}$$

$$\operatorname{rot}\operatorname{rot}\bar{E} = -\frac{i\omega}{c}\mu \operatorname{rot}\bar{H}$$

$$\operatorname{grad}\operatorname{div}\bar{E} - \Delta\bar{E} = -\frac{i\omega}{c}\mu\frac{i\omega}{c}(\varepsilon\bar{E} + \delta^2\operatorname{grad}\operatorname{div}\bar{E})$$

$$\Delta\bar{E} + \left(\frac{\omega^2}{c^2}\mu\delta^2 - 1\right)\operatorname{grad}\operatorname{div}\bar{E} + \frac{\omega^2}{c^2}\varepsilon\mu\bar{E} = 0$$

$$\bar{E} = \bar{z}_0 E_z e^{-ikz}$$

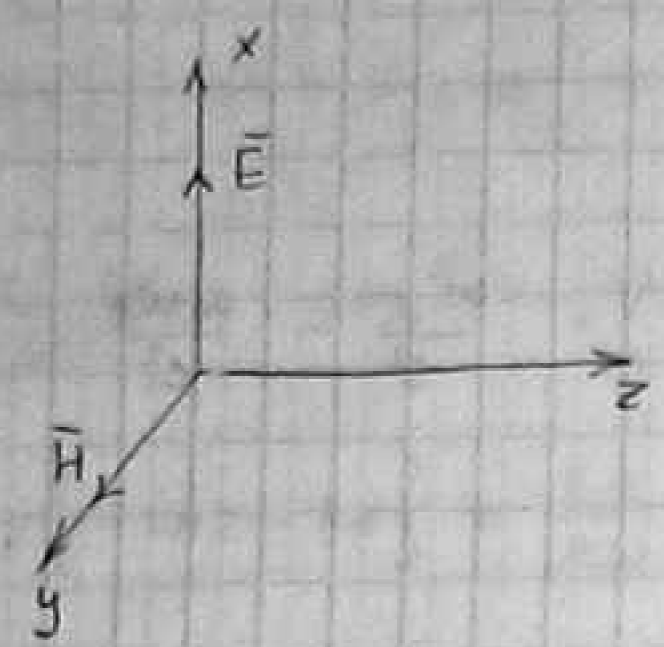
$$-k^2 E_z e^{-ikz} + \left(\frac{\omega^2}{c^2}\mu\delta^2 - 1\right)(-k^2 E_z e^{-ikz}) + \frac{\omega^2}{c^2}\varepsilon\mu E_z e^{-ikz} = 0$$

$$-k^2\frac{\omega^2}{c^2}\mu\delta^2 + \frac{\omega^2}{c^2}\varepsilon\mu = 0, \quad k^2\delta^2 = \varepsilon$$

$$k^2\frac{V_T^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2},$$

$$\omega^2 = \omega_p^2 + k^2 V_T^2$$

N 7.13



Стоячая волна

$$E_x = E_0 e^{-ikz} + E_0 e^{ikz} = 2E_0 \frac{e^{-ikz} + e^{ikz}}{2} =$$

$$= 2E_0 \cos kz$$

$E_x = 2E_0 \cos kz \cos \omega t$ — реальная волна

$$\mathrm{rot}\,\bar E = -\frac{i\omega\mu}{c}\bar H, \quad \bar H = \frac{ic}{\omega\mu}\mathrm{rot}\,\bar E$$

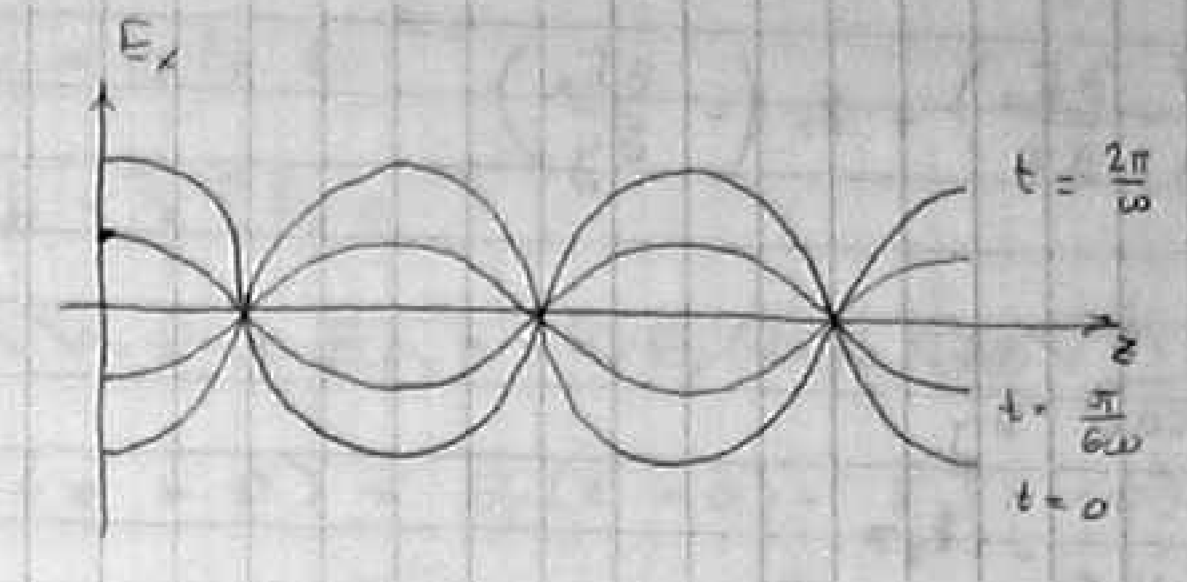
$$\mathrm{rot}\,\bar E = \begin{vmatrix} \bar x_0 & \bar y_0 & \bar z_0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = -\bar y_0\left(-\frac{\partial E_x}{\partial z}\right), \text{ т.к.}$$

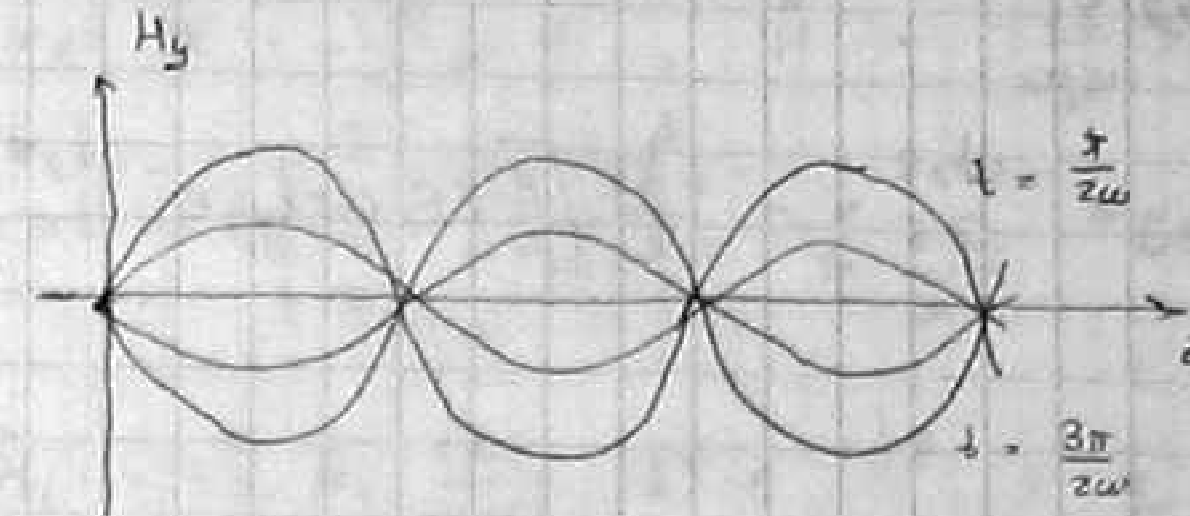
$$H_y = \frac{ic}{\omega\mu}\frac{\partial E_x}{\partial z} = -\frac{ic}{\omega\mu} 2E_0 k \sin kz = \frac{2E_0 kc}{\omega\mu} e^{-i\frac{\pi}{2}} \sin kz =$$

$$= \frac{2E_0}{\mu}\frac{c}{\omega}\frac{\omega}{c}\sqrt{\varepsilon\mu}\, e^{-i\frac{\pi}{2}} \sin kz = 2E_0 \sqrt{\frac{\varepsilon}{\mu}} \sin kz\, e^{-i\frac{\pi}{2}}$$

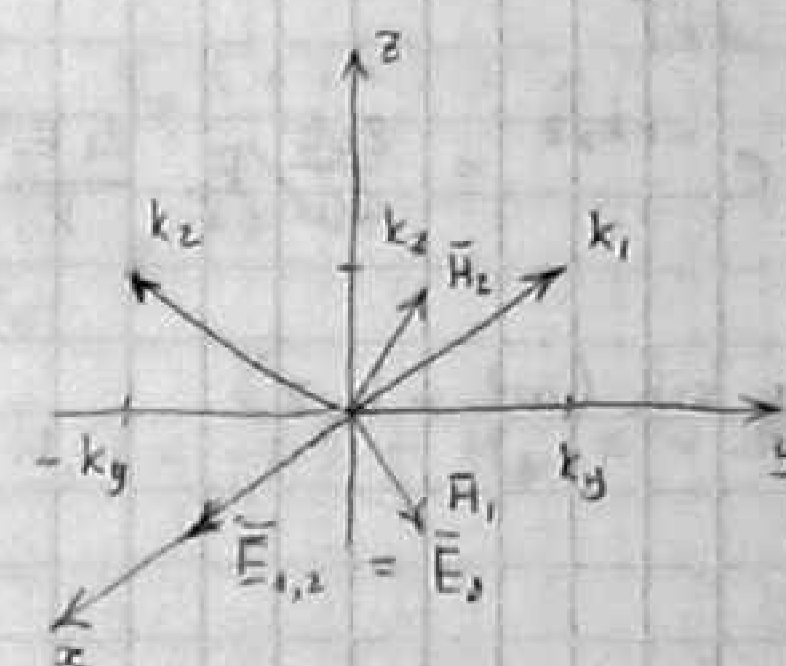
Для реальной волны

$$H_y = 2E_0\sqrt{\frac{\varepsilon}{\mu}} \sin kz \cos\left(\omega t - \frac{\pi}{2}\right) = 2E_0\sqrt{\frac{\varepsilon}{\mu}} \sin kz \sin \omega t$$





N 7.14



$$E_x = E_0 e^{-ik_y y - ik_z z} + E_0 e^{ik_y y - ik_z z} =$$

$$= 2E_0 e^{-ik_z z} \frac{e^{-ik_y y} + e^{ik_y y}}{2} = 2E_0 \cos k_y y\, e^{-ik_z z}$$

$$\mathrm{Re}\{E_x e^{i\omega t}\} = 2E_0 \cos k_y y \cos(\omega t - k_z z)$$

$$\bar H = \frac{ic}{\omega\mu}\mathrm{rot}\,\bar E$$

$$\operatorname{rot} E = \begin{vmatrix} \bar{x}_0 & \bar{y}_0 & \bar{z}_0 \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = -\bar{y}_0\left(-\frac{\partial E_x}{\partial z}\right) + \bar{z}_0\left(-\frac{\partial E_x}{\partial y}\right)$$

$$H_y = \frac{ic}{\omega\mu}\frac{\partial E_x}{\partial z}, \quad H_z = -\frac{ic}{\omega\mu}\frac{\partial E_x}{\partial y}$$

$$H_y = i\frac{ic}{\omega\mu}(-k_z)\, 2E_0 \cos k_y y\, e^{-ik_z z} =$$

$$= -i^2\left(\frac{c}{\omega}\frac{1}{\sqrt{\varepsilon\mu}}\right)\sqrt{\frac{\varepsilon}{\mu}}\, k_z\, 2E_0 \cos k_y y\, e^{-ik_z z} = -\frac{i^2 k_z}{k}\sqrt{\frac{\varepsilon}{\mu}}\, 2E_0 \cos k_y y\, e^{-ik_z z} =$$

$$= \frac{k_z}{k}\sqrt{\frac{\varepsilon}{\mu}}\, 2E_0 \cos k_y y\, e^{-ik_z z}$$

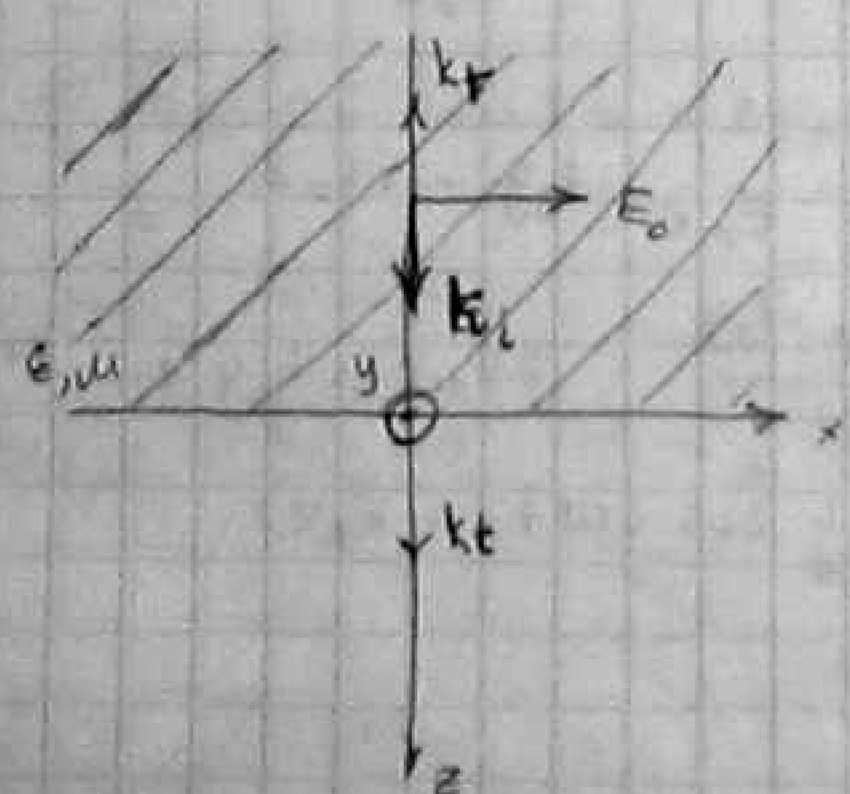
$$\operatorname{Re}\{H_y e^{i\omega t}\} = \frac{k_z}{k}\sqrt{\frac{\varepsilon}{\mu}}\, 2E_0 \cos k_y y \cos(\omega t - k_z z)$$

$$H_z = -\frac{ic}{\omega\mu}(-2E_0 k_y \sin k_y y)\, e^{-ik_z z} = \cancel{\frac{2ic}{\omega\mu}}\cancel{E_0}\ \frac{2iE_0 k_y}{k}\sin k_y y\, e^{-ik_z z}$$

$$\operatorname{Re}\{H_z e^{i\omega t}\} = -\frac{2E_0 k_y}{k}\sin k_y y \sin(\omega t - k_z z)$$

№ 8.1

a)



$$\bar{E}_i = \bar{x}_0 E_0 e^{-ikz}$$

$$\bar{E}_r = \bar{x}_0 \Gamma E_0 e^{+ikz}$$

$$\cancel{\bar{E}_\Sigma = \bar{E}_i + \bar{E}_r = \bar{x}_0 E_0 (1 + \Gamma) e^{-ik}}$$

$$\bar{E}(z=0) = \bar{E}_i(z=0) + \bar{E}_r(z=0) = \bar{x}_0 E_0 (1 + \Gamma)$$

$$\operatorname{rot} \bar{E} = -\frac{i\omega}{c}\mu \bar{H}, \quad \bar{H} = \frac{ic}{\omega\mu}\operatorname{rot}\bar{E}$$

$$\operatorname{rot}\bar{E} = \begin{vmatrix} \bar{x}_0 & \bar{y}_0 & \bar{z}_0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} = \bar{y}_0 \frac{\partial E_x}{\partial z}$$

$$\bar{H}_i = +\bar{y}_0 E_0 \frac{ic}{\omega\mu}(-ik)\, e^{-ikz} = \bar{y}_0 E_0 \frac{c}{\omega\mu}\frac{\omega}{c}\sqrt{\varepsilon\mu}\, e^{-ikz} =$$

$$= \bar{y}_0 E_0 \sqrt{\frac{\varepsilon}{\mu}}\, e^{-ikz}$$

$$\bar{H}_r = \bar{y}_0 \frac{ic}{\omega\mu} E_0 (+ik)\,\Gamma e^{ikz} = -\bar{y}_0 E_0 \frac{c}{\omega\mu}\frac{\omega}{c}\sqrt{\varepsilon\mu}\, e^{ikz} =$$

$$= -\bar{y}_0 \Gamma E_0 \sqrt{\frac{\varepsilon}{\mu}}\, e^{ikz}$$

$$\bar{H}(z=0) = \bar{H}_i(z=0) + \bar{H}_r(z=0) = y_0 E_0 \sqrt{\frac{\varepsilon}{\mu}}(1 - \Gamma)$$

$$\eta_s = \frac{E_x(0)}{H_y(0)}$$

$$\eta_s = \frac{E_0(1+\Gamma)}{E_0\sqrt{\frac{\varepsilon}{\mu}}(1-\Gamma)}; \quad \eta_s\sqrt{\frac{\varepsilon}{\mu}} = \frac{1+\Gamma}{1-\Gamma}; \quad \eta_\omega = \sqrt{\frac{\mu}{\varepsilon}}$$

$$\frac{\eta_s}{\eta_\omega} = \frac{1+\Gamma}{1-\Gamma}, \quad \eta_s - \eta_s\Gamma = \eta_\omega + \eta_\omega\Gamma; \quad \eta_s - \eta_\omega = (\eta_s + \eta_\omega)\Gamma$$

$$\Gamma = \frac{\eta_s - \eta_\omega}{\eta_s + \eta_\omega}$$

б) $\bar{E}_i = \bar{x}_0 E_0 e^{-ikz}$

$\bar{E}_r = \bar{x}_0 \Gamma E_0 e^{ikz}$

$\bar{H}_i = \bar{y}_0 \frac{E_0}{\eta_\omega} e^{-ikz}$

$\bar{H}_r = -\bar{y}_0 \frac{\Gamma E_0}{\eta_\omega} e^{ikz}$

$\eta(L) = \left(\frac{E_x}{E_y}\right)_{z=-L}$

$E_x(z=-L) = E_0 e^{ikL} + \Gamma E_0 e^{-ikL} = E_0\left(e^{ikL} + \frac{\eta_s - \eta_\omega}{\eta_s + \eta_\omega} e^{-ikL}\right) =$

$= \cancel{E_0} \frac{E_0}{\eta_s + \eta_\omega}\left(\eta_s e^{ikL} + \eta_\omega e^{ikL} + \eta_s e^{-ikL} - \eta_\omega e^{-ikL}\right) =$

$= \frac{E_0}{\eta_s + \eta_\omega}\left(\eta_s(e^{ikL} + e^{-ikL}) + \eta_\omega(e^{ikL} - e^{-ikL})\right) =$

$= \frac{E_0}{\eta_s + \eta_\omega}\left(2\eta_s \cos kL + 2i\eta_\omega \sin kL\right)$

$H_y(z=-L) = \frac{1}{\eta_\omega} E_0 e^{ikL} - \frac{1}{\eta_\omega} E_0 \Gamma e^{-ikL} =$

$= \frac{E_0}{\eta_\omega} e^{ikL} - \frac{E_0}{\eta_\omega} \frac{\eta_s - \eta_\omega}{\eta_s + \eta_\omega} e^{-ikL} = \frac{E_0}{\eta_\omega(\eta_s + \eta_\omega)}\left(\eta_s e^{ikL} + \eta_\omega e^{ikL} - \eta_s e^{-ikL} + \eta_\omega e^{-ikL}\right) =$

$= \frac{E_0}{\eta_\omega(\eta_s + \eta_\omega)}\left(2i\eta_s \sin kL + 2\cancel{i}\eta_\omega \cos kL\right)$

$\eta(L) = \eta_\omega \frac{2\eta_s \cos kL + 2i\eta_\omega \sin kL}{2\eta_\omega \cos kL + 2i\eta_s \sin kL} = \eta_\omega \frac{\eta_s + i\eta_\omega \,\mathrm{tg}\, kL}{\eta_\omega + i\eta_s \,\mathrm{tg}\, kL}$

в) $E = E_0 e^{-ikz} + \Gamma E_0 e^{ikz}$; $\Gamma = |\Gamma| e^{i\varphi}$

$|E|^2(z) = E(z) \cdot E^*(z)$

$E^* = E_0 e^{ikz} + E_0 |\Gamma| e^{-i\varphi} e^{-ikz}$

$E \cdot E^* = E_0^2 + |\Gamma| E_0^2 e^{i\varphi} e^{2ikz} + |\Gamma| E_0^2 e^{-i\varphi} e^{-2ikz} + E_0^2 |\Gamma|^2 =$

$= E_0^2 + 2|\Gamma| E_0^2 \frac{e^{i(\varphi + 2kz)} + e^{-i(\varphi + 2kz)}}{2} + E_0^2 |\Gamma|^2 =$

$= E_0^2 (1 + 2|\Gamma| \cos(\varphi + 2kz) + |\Gamma|^2)$

$|E|^2_{max}$ если $\cos(\varphi + 2kz) = 1$:

$|E|^2_{max} = E_0^2 (1 + 2|\Gamma| + |\Gamma|^2) = E_0^2 (1 + |\Gamma|)^2$

$|E|^2_{min}$ если $\cos(\varphi + 2kz) = -1$:

$|E|^2_{min} = E_0^2 (1 - 2|\Gamma| + |\Gamma|^2) = E_0^2 (1 - |\Gamma|)^2$

$КСВ = \frac{|E|^2_{max}}{|E|^2_{min}} = \left(\frac{1 + |\Gamma|}{1 - |\Gamma|}\right)^2$

г) $КСВ = 1$, значит $1 + |\Gamma| = 1 - |\Gamma|$, $|\Gamma| = -|\Gamma| \Rightarrow$

$|\Gamma| = 0$

$\Gamma = \frac{\eta_s - \eta_\omega}{\eta_s + \eta_\omega} \Rightarrow \eta_s = \eta_\omega = \sqrt{\frac{\mu}{\varepsilon}}$

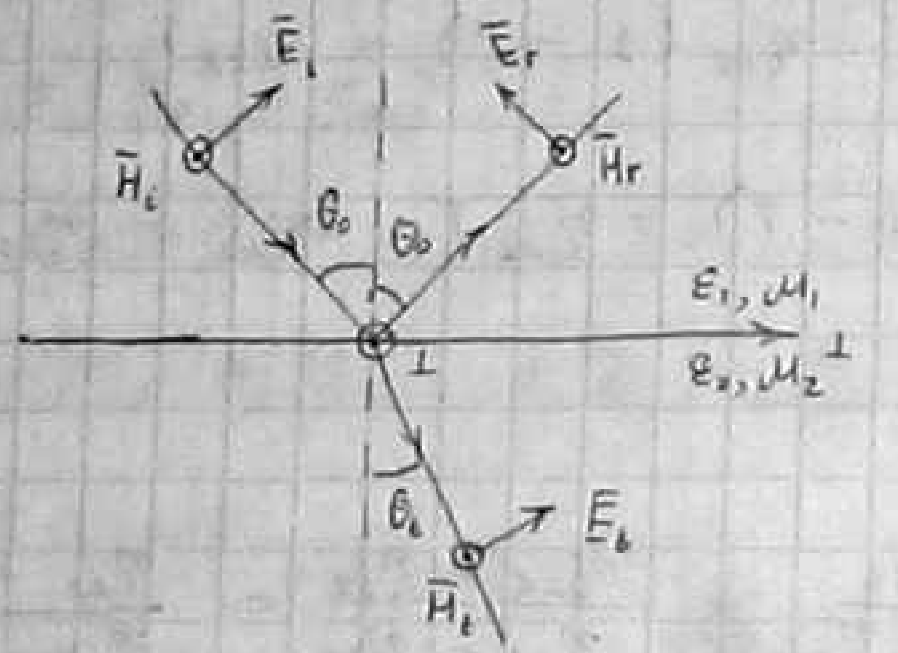
$КСВ = \infty$, значит $1 - |\Gamma| = 0$, $|\Gamma| = 1$

$\Gamma = \frac{\eta_s - \eta_\omega}{\eta_s + \eta_\omega}$, при $\eta_s = \infty$ $\Gamma = \lim_{\eta_s \to \infty} \frac{\eta_s - \eta_\omega}{\eta_s + \eta_\omega} =$

$= \lim_{\eta_s \to \infty} \frac{\eta_s}{\eta_s} = 1$

$$\eta_{\perp 2} = \sqrt{\frac{\mu_2}{\varepsilon_2}}\left(1 - \frac{\varepsilon_1\mu_1}{\varepsilon_2\mu_2}\sin^2\theta_0\right)^{-1/2}$$

б) TM



$H_{\perp i} = \frac{E_{\perp i}}{\eta_{\perp 1}}$, $H_{\perp r} = +\frac{E_{\perp r}}{\eta_{\perp 1}}$, $H_{\perp t} = \frac{E_{\perp t}}{\eta_{\perp 2}}$

$E_{\perp i} = E_{0\perp}$; $E_{\perp r} = -\Gamma E_{0\perp}$; $E_{\perp t} = T E_{0\perp}$

$H_{\perp 1} = H_{\perp 2}$, $\frac{E_{0\perp} + \Gamma E_{0\perp}}{\eta_{\perp 1}} = \frac{T E_{0\perp}}{\eta_{\perp 2}}$

$E_{\perp 1} = E_{\perp 2}$, $E_{0\perp} - \Gamma E_{0\perp} = T E_{0\perp}$

$$\begin{cases} \eta_{\perp 2}(1+\Gamma) = T\eta_{\perp 1} \\ 1 - \Gamma = T \end{cases}$$

$\Gamma = 1 - T$, $\eta_{\perp 2}(2 - T) = T\eta_{\perp 1}$;

$H_{\perp i} = H_0$; $H_{\perp r} = \Gamma H_0$; $H_{\perp t} = T H_0$

$E_{\perp i} = \eta_{\perp 1} H_{\perp 1} = \eta_{\perp 1} H_0$; $E_{\perp r} = -\eta_{\perp 1} H_{\perp r} = -\eta_{\perp 1}\Gamma H_0$

$E_{\perp t} = \eta_{\perp 2} H_{\perp t} = \eta_{\perp 2} T H_0$

$H_{\perp 1} = H_{\perp 2}$, $E_{\perp 1} = E_{\perp 2}$

$1 + \Gamma = T$

$\eta_{\perp 1} - \eta_{\perp 1}\Gamma \cancel{H_0} = T\eta_{\perp 2}\cancel{H_0}$

$\Gamma = T - 1$; & $2\eta_{\perp 1} - \eta_{\perp 1} T = T\eta_{\perp 2}$;

$$T = \frac{2\eta_{\perp 1}}{\eta_{\perp 1}+\eta_{\perp 2}}$$

$$\Gamma = \frac{2\eta_{\perp 1}}{\eta_{\perp 1}+\eta_{\perp 2}} - \frac{\eta_{\perp 1}+\eta_{\perp 2}}{\eta_{\perp 1}+\eta_{\perp 2}} = \frac{\eta_{\perp 1}-\eta_{\perp 2}}{\eta_{\perp 1}+\eta_{\perp 2}}$$

Для первой среды

$E_0 = \eta_1 H_0$; $H_\perp = H_0$, $E_\perp = E_0\cos\theta_0$

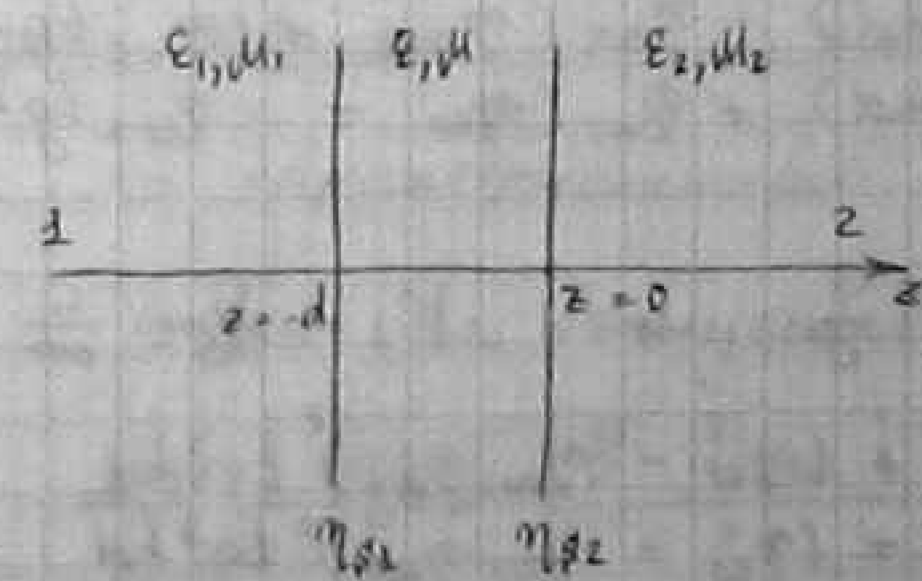
$$\eta_\perp = \frac{E_\perp}{H_\perp} = \frac{\eta_1 H_0\cos\theta_0}{H_0} = \sqrt{\frac{\mu_1}{\varepsilon_1}}\cos\theta_0$$

Для второй среды

$E_0 = \eta_2 H_0$; $H_\perp = H_0$, $E_\perp = E_0\cos\theta_t$

$$\eta_\perp = \sqrt{\frac{\mu_2}{\varepsilon_2}}\cos\theta_t = \sqrt{\frac{\mu_2}{\varepsilon_2}}\left(1-\frac{\varepsilon_1\mu_1}{\varepsilon_2\mu_2}\sin^2\theta_0\right)^{1/2}$$

№8,3



Т.к. в области 2 существует плоская волна движущаяся по z, для нее $\eta_{w2} = \sqrt{\frac{\mu_2}{\varepsilon_2}}$

На границе $\eta_{\delta 2} = \eta_{w2} = \sqrt{\frac{\mu_2}{\varepsilon_2}}$.

Формула пересчета импеданса

$$\eta(d) = \eta_w \frac{\eta_{\delta 2} + i\eta_w \,tg\, kd}{\eta_w + i\eta_{\delta 2}\,tg\, kd}, \text{ где}$$

$$\eta_w = \sqrt{\frac{\mu}{\varepsilon}}, \quad k = \frac{\omega}{c}\sqrt{\varepsilon\mu}$$

На границе слоя со средой 1 $\eta_{\delta 1} = \eta(d)$

$$\eta_{\delta 2} = \eta_w \frac{\eta_{\delta 2} + i\eta_w \,tg\, kd}{\eta_w + i\eta_{\delta 2}\,tg\, kd} = \eta_w \frac{\eta_{w2} + i\eta_w \,tg\, kd}{\eta_w + i\eta_{w2}\,tg\, kd}$$

Для коэффициента Г:

$$\Gamma = \frac{\eta_{\delta 1} - \eta_{w1}}{\eta_{\delta 1} + \eta_{w1}}, \text{ где } \eta_{w1} = \sqrt{\frac{\mu_1}{\varepsilon_1}}.$$

$$\Gamma = \frac{\eta_w \frac{\eta_{w2} + i\eta_w \,tg\, kd}{\eta_w + i\eta_{w2}\,tg\, kd} - \eta_{w1}}{\eta_w \frac{\eta_{w2} + i\eta_w \,tg\, kd}{\eta_w + i\eta_{w2}\,tg\, kd} + \eta_{w1}} =$$

$$= \frac{\eta_w(\eta_{w2} + i\eta_w \,tg\, kd) - \eta_{w1}(\eta_w + i\eta_{w2}\,tg\, kd)}{\eta_w(\eta_{w2} + i\eta_w \,tg\, kd) + \eta_{w1}(\eta_w + i\eta_{w2}\,tg\, kd)}$$

$\Gamma = 0$, если $|\Gamma| = \frac{0}{c}$ либо $|\Gamma| = \frac{c}{\infty}$

$$|\Gamma|^2 = \frac{\eta_w^2(\eta_{w2} - \eta_{w1})^2 + (\eta_w^2 - \eta_{w1}\eta_{w2})^2 tg^2 kd}{\eta_w^2(\eta_{w2} + \eta_{w1})^2 + (\eta_w^2 + \eta_{w1}\eta_{w2})^2 tg^2 kd}$$

$\eta_{w2} \neq \eta_{w1} \Rightarrow \eta_w^2(\eta_{w2} - \eta_{w1})^2 + (\eta_w^2 - \eta_{w1}\eta_{w2})^2 tg^2 kd \neq 0$

Отношение $\Gamma = \frac{0}{c}$ невозможно. Чтобы было возможно отношение $\Gamma = \frac{c}{\infty}$, необходимо $(\eta_w^2 - \eta_{w1}\eta_{w2})^2 = 0$,

т.е. $\eta_w = \sqrt{\eta_{w1}\eta_{w2}}$ и $tg^2 kd = \infty$, $kd = \frac{\pi}{2}(2n+1)$, где $n = 0, 1, 2, \dots$, в итоге при $\eta_{w2} \neq \eta_{w1}$:

$kd = \frac{\pi}{2}(2n+1)$, $n = 0, 1, 2, \dots$ и $\eta_w = \sqrt{\eta_{w1}\eta_{w2}}$

Если $\eta_{w2} = \eta_{w1}$, то предел $\Gamma = \frac{c}{\infty}$, невозможен, т.к. $\eta_w^2(\eta_{w2} - \eta_{w1})^2 + (\eta_w^2 - \eta_{w1}\eta_{w2})^2 tg^2 kd =$

$= (\eta_w^2 - \eta_{w1}\eta_{w2})^2 tg^2 kd$

Рассмотрим отношение $\Gamma = \frac{0}{c}$, т.е. $(\eta_w^2 - \eta_{w1}\eta_{w2}) = 0$ либо $tg\, kd = 0$:

$tg\, kd = 0 \Rightarrow kd = \frac{\pi}{4}(2n+1)$, где $n = 0, 1, 2, \dots$

$\eta_w^2 - \eta_{w1}\eta_{w2} = 0 \Rightarrow \eta_w = \eta_{w1}$

№8,4

$|\Gamma|$ не будет зависеть от толщины слоя d, если этот слой а по характеристикам является первой или второй средой, т.е. получается нормальное падение на границу двух сред. В итоге должно быть

$\eta_w = \eta_{w1}$, либо $\eta_w = \eta_{w2}$

~~xxx~~
N 9.1

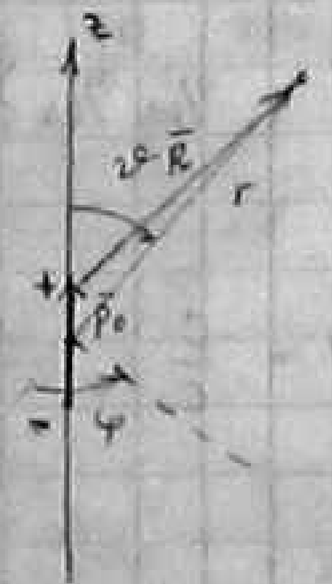
a) $\bar P = \bar p_0 \exp\{i\omega t\}$; $|\bar P| = q|\bar l|$, $ql = p_0 \exp\{i\omega t\}$

$\dot q = i\omega \frac{p_0}{l} \exp\{i\omega t\}$, $kl \ll 1 \Rightarrow$

$\bar j = \dot q\, \delta(x)\delta(y) = -i\omega \frac{\bar p_0}{l} \exp\{i\omega t\}\, \delta(x)\delta(y)$

$\mu = 1;\ \varepsilon = 1$

$$\bar A = \frac{1}{c}\int_{V'} \bar j \frac{e^{-ikR}}{R}\, dV'$$



$\bar p_0 = p_0 \bar z_0$

$kl \ll 1$ $\quad r \gg l \Rightarrow R \approx r \Rightarrow$

$$\bar A = -\frac{1}{c}\int_{-l/2}^{l/2} i\omega \frac{p_0 \bar z_0}{l} \frac{e^{-ikr}}{r}\, dz = -\frac{i\omega p_0 \bar z_0}{lc}\frac{e^{-ikr}}{r}\int_{-l/2}^{l/2} dz =$$

$$= -\frac{i\omega p_0 \bar z_0}{lc}\cdot\frac{e^{-ikr}}{r}\, l = -\bar z_0 \frac{i\omega}{cr} p_0 e^{-ikr}$$

$\bar H = \bar B = \mathrm{rot}\, \bar A$

$$\mathrm{rot}\, \bar A = \begin{vmatrix} \bar x_0 & \bar y_0 & \bar z_0 \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & A_z \end{vmatrix} = \bar x_0 \frac{\partial A_z}{\partial y} - \bar y_0 \frac{\partial A_z}{\partial x} =$$

$$= \bar x_0 \frac{\partial A_z}{\partial r}\frac{\partial r}{\partial y} - \bar y_0 \frac{\partial A_z}{\partial r}\frac{\partial r}{\partial x} = \frac{\partial A_z}{\partial r}\left(\bar x_0 \frac{\partial r}{\partial y} - \bar y_0 \frac{\partial r}{\partial x}\right)$$

$\frac{\partial r}{\partial x} = \frac{x}{r}$, $\frac{\partial r}{\partial y} = \frac{y}{r}$, $\bar H = \frac{\partial A_z}{\partial r}\frac{\bar x_0 y - \bar y_0 x}{r} =$

$$= \frac{\partial A_z}{\partial r}\frac{\bar x_0 r \sin\vartheta \sin\varphi - \bar y_0 r \sin\vartheta \cos\varphi}{r} =$$

$$= \frac{\partial A_z}{\partial r}\sin\vartheta\,(\bar x_0 \sin\varphi - \bar y_0 \cos\varphi) = \frac{\partial A_z}{\partial r}\sin\vartheta\, \bar\varphi_0$$

$$\frac{\partial A_z}{\partial r} = -\frac{i\omega p_0}{c}\left(-\frac{1}{r^2} - \frac{ik}{r}\right)e^{-ikr} = -ikp_0\left(-\frac{1}{r^2} - \frac{ik}{r}\right)e^{-ikr}$$

$$H_\varphi = +ikp_0 \frac{k}{r}\left(\frac{1}{kr} + i\right)e^{-ikr}\sin\vartheta =$$

$$= +\frac{k^2 p_0}{r}\left(\frac{i}{kr} - 1\right)e^{-ikr}\sin\vartheta$$

$\mathrm{rot}\,\bar H = \frac{1}{c}\cancel{\frac{\partial \bar D}{\partial t}}\, i\omega \bar D = \frac{i\omega}{c}\bar E$, $\bar E = -\frac{ic}{\omega}\mathrm{rot}\,\bar H$

$$\mathrm{rot}\,\bar H = \begin{vmatrix} \bar r_0 / r^2 \sin\vartheta & \bar\varphi_0 / r & \bar\vartheta_0 / r\sin\vartheta \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial \vartheta} \\ 0 & r\sin\vartheta\, H_\varphi & 0 \end{vmatrix} =$$

$$= -\frac{\bar r_0}{r^2 \sin\vartheta}\frac{\partial(r\sin\vartheta\, H_\varphi)}{\partial\vartheta} + \frac{\bar\vartheta_0}{r\sin\vartheta}\frac{\partial(r\sin\vartheta\, H_\varphi)}{\partial r}$$

$$E_r = \frac{ic}{\omega}\frac{r}{r^2 \sin\vartheta} k^2 p_0 \frac{1}{r}\left(\frac{i}{kr} - 1\right)e^{-ikr}\frac{\partial(\sin^2\vartheta)}{\partial\vartheta} =$$

$$= \frac{ic}{\omega}\frac{k^2 p_0}{r^2 \sin\vartheta}\left(\frac{i}{kr} - 1\right)e^{-ikr}\, 2\sin\vartheta\cos\vartheta =$$

$$= -\frac{2kp_0}{r^2}\cos\vartheta\left(\frac{1}{kr} + i\right)e^{-ikr}$$

$$E_\vartheta = -\frac{ic1}{\omega r}\frac{\partial (rH_\varphi)}{\partial r} = -\frac{ic1}{\omega r}\frac{\partial}{\partial r}\left(r\frac{k^2p_0}{r}\left(\frac{i}{kr}-1\right)e^{-ikr}\sin\vartheta\right) =$$

$$= -\frac{i}{kr}k^2p_0\sin\vartheta\,\frac{\partial}{\partial r}\left(\left(\frac{i}{kr}-1\right)e^{-ikr}\right) =$$

$$= -\frac{i}{r}kp_0\sin\vartheta\left[-\frac{i}{kr^2} - ik\left(\frac{i}{kr}-1\right)\right]e^{-ikr} =$$

$$= -\frac{k^2p_0}{r}\sin\vartheta\left(-1+\frac{i}{kr}+\frac{1}{k^2r^2}\right)e^{-ikr}$$

При $kr \ll 1 \Rightarrow \frac{1}{kr} \gg 1 \Rightarrow$

$$H_\varphi = \frac{ik^2p_0}{r}\frac{1}{kr}e^{-ikr}\sin\vartheta = \frac{ikp_0}{r^2}\sin\vartheta$$

$$E_r = -\frac{2kp_0}{r^2}\cos\vartheta\frac{1}{kr} = \cancel{-\frac{2p_0}{}} - \frac{2p_0}{r^3}\cos\vartheta$$

$$E_\vartheta = -\frac{k^2p_0}{r}\sin\vartheta\frac{1}{k^2r^2} = -\frac{p_0}{r^3}\sin\vartheta$$

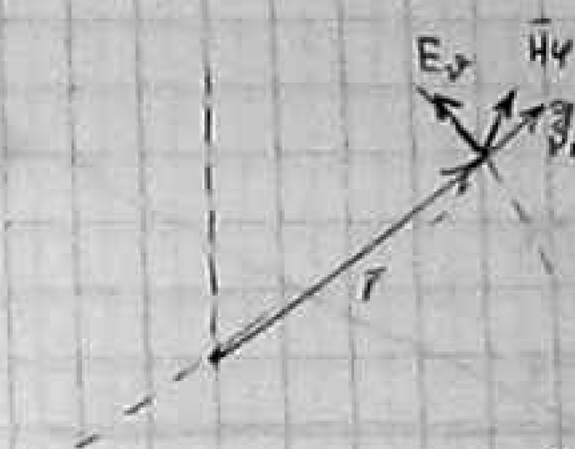
При $kr \gg 1 \Rightarrow \frac{1}{kr} \ll 1 \Rightarrow$

$$H_\varphi = \frac{k^2p_0}{r}(-1)e^{-ikr} = -\frac{k^2p_0}{r}e^{-ikr}$$

$$E_r = -\frac{2kp_0}{r^2}i\cos\vartheta\, e^{-ikr} = -\frac{2ikp_0}{r^2}\cos\vartheta\, e^{-ikr}$$

$$E_\vartheta = -\frac{k^2p_0}{r}\sin\vartheta(-1)e^{-ikr} = \frac{k^2p_0}{r}\sin\vartheta\, e^{-ikr}$$

б) $\tilde{S} = \mathrm{Re}\,\frac{c}{8\pi}[\bar E \times \bar H^*]$



$$\tilde S_r = -\mathrm{Re}\,\frac{c}{8\pi}E_\vartheta H_\varphi^* = +\mathrm{Re}\,\frac{c}{8\pi}\frac{k^2p_0}{r}\sin\vartheta\left(-1+\frac{i}{kr}+\frac{1}{k^2r^2}\right)e^{-ikr}\cdot$$

$$\cdot\frac{k^2p_0}{r}\left(-\frac{i}{kr}-1\right)e^{ikr}\sin\vartheta =$$

$$= \frac{c}{8\pi}\frac{k^4p_0^2}{r^2}\sin^2\vartheta\;\mathrm{Re}\left\{\frac{i}{kr}+1+\frac{1}{k^2r^2}-\frac{i}{kr}-\frac{i}{k^3r^3}-\frac{1}{k^2r^2}\right\} =$$

$$= \frac{ck^4p_0^2}{8\pi r^2}\sin^2\vartheta\;\mathrm{Re}\left\{1-\frac{i}{k^3r^3}\right\} = \frac{ck^4p_0^2}{8\pi r^2}\sin^2\vartheta$$

в) $$P = \int_S \tilde S_r\,dS = \int_0^{2\pi}\int_0^{\pi}\frac{ck^4p_0^2}{8\pi r^2}\sin^2\vartheta\; r\,d\vartheta\; r\sin\vartheta\,d\varphi =$$

$$= \frac{ck^4p_0^2}{8\pi}\int_0^{2\pi}d\varphi\int_0^{\pi}\sin^2\vartheta\,\sin\vartheta\,d\vartheta = -\frac{ck^4p_0^2}{4}\int_0^{\pi}(1-\cos^2\vartheta)\,d(\cos\vartheta) =$$

$$= -\frac{ck^4p_0^2}{4}\left(\cos\vartheta-\frac{\cos^3\vartheta}{3}\right)\Big|_0^{\pi} = -\frac{ck^4p_0^2}{4}\left(-1-1+\frac{1}{3}+\frac{1}{3}\right) =$$

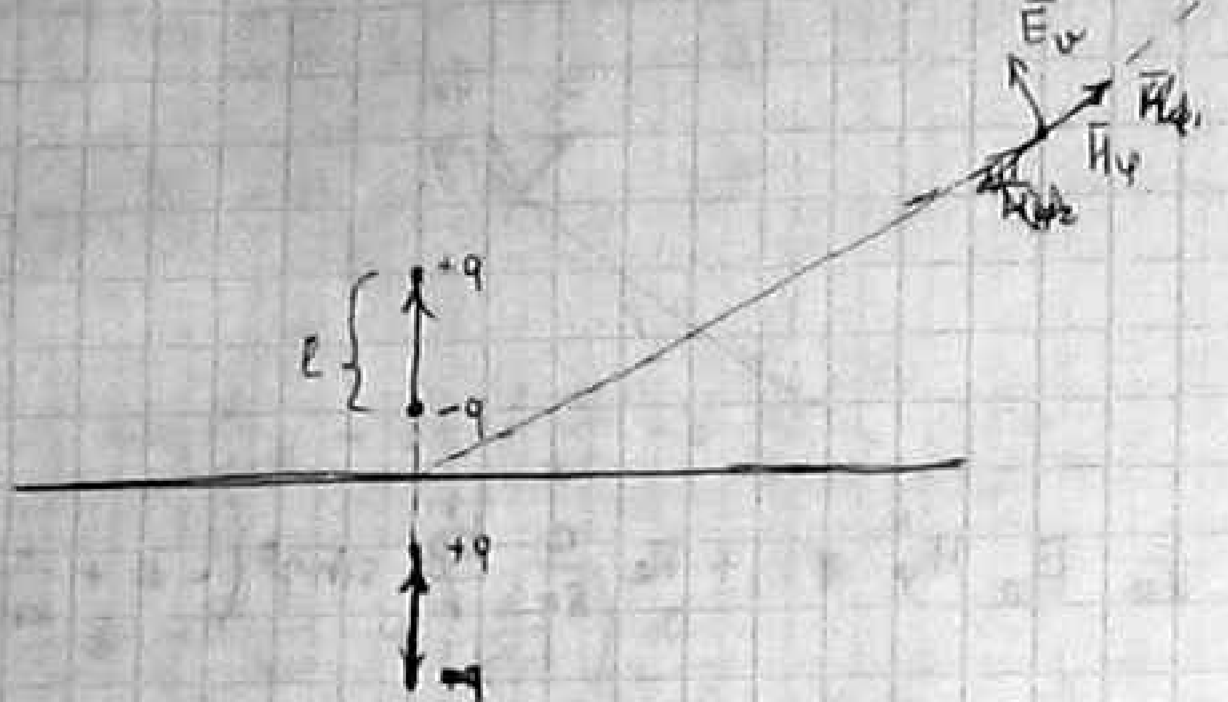
$$= -\frac{ck^4p_0^2}{4}\left(-\frac{4}{3}\right) = \frac{ck^4p_0^2}{3} = \frac{c\omega^4p_0^2}{3c^4} = \frac{\omega^4p_0^2}{3c^3}$$

$$I_0 = \frac{\omega p_0}{\ell},\quad I_0^2 = \frac{\omega^2p_0^2}{\ell^2},$$

$$R_r^{(e)} = \frac{2\omega^4p_0^2}{3c^3}\cdot\frac{\ell^2}{\omega^2p_0^2} = \frac{2\omega^2\ell^2}{3c^3} = \frac{2(k\ell)^2}{3c}$$

№9.6

а)



Поле элементарного вибратора дв. длины 2l.

$F \sim \sin^2 2\theta$

б)