Методы математической физики
Контрольная по вариационному исчислению

Билет № 1.

1. $\int\limits_0^1 \ddot{x}_2^2\,dt \to \text{extr},\ \dot{x}_1 = x_2,\ x_1(0) = x_2(0) = 0,\ x_1(1)$.

2. $\begin{cases} \int_0^\pi \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 x^2\,dt = 1, \\ x(0) = x(1) = 0. \end{cases}$

3. $\int_0^\pi \dot{x}^2\,dt \to \text{extr},\ \int_0^\pi x\sin(t)\,dt = 0,\ x(0) = 0,\ x(\pi) = 1.$

4. $\begin{cases} \int_0^\pi \left(\ddot{x}^2 - \dot{x}^2\right)\,dt \to \text{extr}, \\ x(0) = 0,\ x(\pi) = 1, \\ \dot{x}(0) = 0,\ \dot{x}(\pi) = 0. \end{cases}$

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Билет № 2.

1. $\begin{cases} \int_0^{t_1} \left(\dot{x}^2 + x\right)\,dt \to \text{extr}, \\ x(0) = 1\,. \end{cases}$

2. $\int\limits_0^T \left(\dot{x}^2 - x + 1\right)\,dt \to \text{extr},\ x(0) = 0.$

3. $\int_0^1 \left(\dot{x}^2 + x\right)\,dt \to \text{extr},\ x(0) = 1.$

4. $\int_0^1 \left(\dot{x}^2 + x\right)\,dt \to \text{extr},\ x(0) = 1.$

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Билет № 3.

1. $\int_0^1 \dot{x}^2\,dt \to \text{extr},\ \int_0^1 x\,dt = 1,\ \int_0^1 tx\,dt = 0,\ x(0) = x(1) = 0.$

2. $\int\limits_0^T \left(\dot{x}^2 - x + 1\right)\,dt \to \text{extr},\ x(0) = 0.$

3. $\begin{cases} \int_0^1 \left(\dot{x}_1\dot{x}_2 + 6x_1 t + 12x_2 t^2\right)\,dt \to \text{extr}, \\ x_1(0) = x_2(0) = 0, \\ x_1\left(\frac{\pi}{2}\right) = 1,\ x_2\left(\frac{\pi}{2}\right) = -1. \end{cases}$

4. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x\right)\,dt \to \text{extr}, \\ x(1) = 0. \end{cases}$

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Билет № 4.

1. $\begin{cases} \int_0^1 u^2\,dt \to \text{extr}, \\ \ddot{x} - x = 3u, \\ x(0) = 0,\ x(1) = 1, \\ \dot{x}(0) = 0,\ \dot{x}(1) = 1. \end{cases}$

2. $\int\limits_0^{\pi/2} \left(\dot{x}^2 - x^2 - 2x\right)\,dt - 2x^2(0) - x^2\left(\tfrac{\pi}{2}\right) \to \text{extr}.$

3. $\int\limits_0^1 x^2\dot{x}^2\,dt \to \text{extr},\ x(0) = 1,\ x(1) = \sqrt{2}.$

4. $\begin{cases} \int_0^1 u^2\,dt \to \text{extr}, \\ \ddot{x} - x = 3u, \\ x(0) = 0,\ x(1) = 1, \\ \dot{x}(0) = 0,\ \dot{x}(1) = 1. \end{cases}$

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Билет № 5.

1. $\int_0^1 u^2\,dt \to \text{extr},\ \ddot{x} - x = u,\ x(0) = \dot{x}(0) = 0,\ x(1) = \text{sh}(1),\ \dot{x}(1) = \text{ch}(1) + \text{sh}(1)$.

2. $\begin{cases} \int_1^2 t^3\dot{x}^2\,dt \to \text{extr}, \\ \int_1^2 x\,dt = 2, \\ x(1) = 4,\ x(2) = 1. \end{cases}$

3. $\int_0^{\pi/2} u^2\,dt \to \text{extr},\ \ddot{x} + x = u,\ x\left(\frac{\pi}{2}\right) = \dot{x}(0) = 0,\ x(0) = 1, \dot{x}\left(\frac{\pi}{2}\right) = 1.$

4. $\begin{cases} \int_1^3 \left(t^2\dot{x}^2 + t\right)\,dt \to \text{extr}, \\ x(1) = 3,\ x(3) = 0. \end{cases}$

Методы математической физики
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Билет № 6.

1. $\begin{cases} \int_0^\pi \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 x^2\,dt = 1, \\ x(0) = x(1) = 0. \end{cases}$

2. $\int\limits_0^T \left(\dot{x}^2 - x + 1\right)\,dt \to \text{extr},\ x(0) = 0.$

3. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x^2 + 4x\,\text{sh}\,t\right)\,dt \to \text{extr}, \\ x(0) = 0. \end{cases}$

4. $\begin{cases} \int_0^{t_1} \left(\dot{x}^2 + x\right)\,dt \to \text{extr}, \\ x(0) = 1\,. \end{cases}$

Билет № 7.

1. $\int\limits_0^2 \left(\dot{x}^2 - x\right)\, dt \to \text{extr},\ x\,(0) = 0.$

2. $\begin{cases} \int_0^1 u^2\, dt \to \text{extr}, \\ \ddot{x} - x = u, \\ x\,(0) = \dot{x}\,(0) = 0, \\ x\,(1) = \operatorname{sh} 1, \\ \dot{x}\,(1) = \operatorname{sh} 1 + \operatorname{ch} 1. \end{cases}$

3. $\begin{cases} \int_1^3 \left(t^2\dot{x}^2 + t\right)\, dt \to \text{extr}, \\ x\,(1) = 3,\ x\,(3) = 0. \end{cases}$

4. $\begin{cases} \int_0^1 \left(x^2 + u^2\right)\, dt \to \text{extr}, \\ \ddot{x} - 4x = u, \\ x\,(1) = \dot{x}\,(0) = 0, \\ x\,(0) = 0,\ \dot{x}\,(1) = 1. \end{cases}$

Билет № 8.

1. $\int_1^e \left(2x - t^2\dot{x}^2\right)\, dt \to \text{extr},\ x\,(1) = e,\ x\,(e) = 0.$

2. $\int\limits_0^T \sqrt{1 + \dot{x}^2} dt \to \text{extr}, x\,(0) = 0,\ T^2 x\,(T) = 1.$

3. $\begin{cases} \int_0^1 \left(x^2 + u^2\right)\, dt \to \text{extr}, \\ \ddot{x} - x = u, \\ x\,(1) = \dot{x}\,(0) = 0, \\ x\,(0) = 1, \dot{x}\,(1) = 0. \end{cases}$

4. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x\right)\, dt \to \text{extr}, \\ x\,(1) = 0. \end{cases}$

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Билет № 9.

1. $\int_0^1 \dot{x}^2\,dt \to \text{extr}$, $\int_0^1 x\,dt = 1$, $\int_0^1 tx\,dt = 0$, $x(0) = x(1) = 0$.

2. $\int\limits_0^2 \left(\dot{x}^2 - x\right)\,dt \to \text{extr}$, $x(0) = 0$.

3. $\int_0^1 u^2\,dt \to \text{extr}$, $\ddot{x} - x = u$, $x(0) = \dot{x}(0) = 0$, $x(1) = \text{sh}(1)$, $\dot{x}(1) = \text{ch}(1) + \text{sh}(1)$.

4. $\begin{cases} \int_1^2 t^2\dot{x}^2\,dt \to \text{extr}, \\ x(1) = 3,\ x(2) = 1. \end{cases}$

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Билет № 10.

1. $\int_0^1 \dot{x}^2\,dt \to \text{extr}$, $\int_0^1 x\,dt = 1$, $\int_0^1 tx\,dt = 0$, $x(0) = x(1) = 0$.

2. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x^2\right)\,dt \to \text{extr}, \\ \int_0^1 xe^t\,dt = \frac{e^2+1}{4}, \\ x(0) = 0,\ x(1) = e. \end{cases}$

3. $\begin{cases} \int_0^{\pi/2} u^2\,dt \to \text{extr}, \\ \ddot{x} + x = u, \\ x\left(\frac{\pi}{2}\right) = \dot{x}(0) = 0, \\ x(0) = 1,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

4. $\begin{cases} \int_0^{\pi/2} u^2\,dt \to \text{extr}, \\ \ddot{x} + x = u, \\ x\left(\frac{\pi}{2}\right) = \dot{x}(0) = 0, \\ x(0) = 1,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 11.

1. $\int_0^1 \dot{x}^2\,dt \to \text{extr}$, $\int_0^1 x\,e^{-t}\,dt = e$, $x(0) = 2e+1$, $x(1) = 2$.

2. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x^2\right)\,dt \to \text{extr}, \\ \int_0^1 xe^t\,dt = \frac{e^2+1}{4}, \\ x(0) = 0,\ x(1) = e. \end{cases}$

3. $\int_0^{t_1} \left(\dot{x}^2 + x\right)\,dt \to \text{extr}$, $x(t_1) = t_1$.

4. $\int_0^{\pi} \dot{x}^2\,dt \to \text{extr}$, $\int_0^{\pi} x\sin(t)\,dt = 0$, $x(0) = 0$, $x(\pi) = 1$.

Методы математической физики
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Билет № 12.

1. $\begin{cases} \int_0^1 \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 tx\,dt = 0, \\ x(0) = -4,\ x(1) = 4. \end{cases}$

2. $\int_0^1 u^2\,dt \to \text{extr}$, $\ddot{x} - 2x = u$, $x(0) = 0$, $\dot{x}(0) = 1$, $x(1) = \text{sh}(1)\,, \dot{x}(1) = \text{ch}(1) + \text{sh}(1)\,.$

3. $\int_0^1 \dot{x}^2\,dt \to \text{extr}$, $\int_0^1 x\,dt = 1$, $\int_0^1 tx\,dt = 0$, $x(0) = x(1) = 0$.

4. $\int_0^1 u^2\,dt \to \text{extr}$, $\dot{x} + x = 2u$, $x(0) = 1$, $x(1) = 0$.

Билет № 13.

1. $\begin{cases} \int_0^1 \left(\dot{x}_1\dot{x}_2 + 6x_1 t + 12x_2 t^2\right)\, dt \to \text{extr}, \\ x_1(0) = x_2(0) = 0, \\ x_1\left(\frac{\pi}{2}\right) = 1,\ x_2\left(\frac{\pi}{2}\right) = -1. \end{cases}$

2. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x^2\right)\, dt \to \text{extr}, \\ \int_0^1 xe^{-t}\, dt = \left(1 - 3e^{-2}\right)/4, \\ x(0) = 0,\ x(1) = 1/e. \end{cases}$

3. $\begin{cases} \int_0^{t_1} \left(\dot{x}^2 + x\right)\, dt \to \text{extr}, \\ x(t_1) = t_1. \end{cases}$

4. $\begin{cases} \int_1^e (t+1)\, t\ddot{x}^2\, dt \to \text{extr}, \\ x(1) = 0,\ x(e) = e, \\ \dot{x}(1) = 1,\ \dot{x}(e) = 2. \end{cases}$

Билет № 14.

1. $\int_1^e \left(2x - t^2\dot{x}^2\right)\, dt \to \text{extr},\ x(1) = e,\ x(e) = 0.$

2. $\begin{cases} \int_0^1 \dot{x}^2\, dt \to \text{extr}, \\ \int_0^1 tx\, dt = 0, \\ x(0) = -4,\ x(1) = 4. \end{cases}$

3. $\int\limits_0^1 \ddot{x}^2\, dt \to \text{extr},\ x(0) = \dot{x}(0) = x(1) = 0,\ \dot{x}(0) = 1.$

4. $\begin{cases} \int_0^1 \dot{x}^2\, dt \to \text{extr}, \\ \int_0^1 tx\, dt = 0, \\ x(0) = -4,\ x(1) = 4. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 15.

1. $\begin{cases} \int_0^\pi \left(\ddot{x}^2 - \dot{x}^2\right)\, dt \to \text{extr}, \\ x\left(0\right) = 0,\, x\left(\pi\right) = 1, \\ \dot{x}\left(0\right) = 0,\, \dot{x}\left(\pi\right) = 0. \end{cases}$

2. $\int_0^1 \left(\dot{x}^2 + x\right)\, dt \to \text{extr},\, x\left(0\right) = 1.$

3. $\begin{cases} \int_0^\pi \dot{x}^2\, dt \to \text{extr}, \\ \int_0^1 x^2\, dt = 1, \\ x\left(0\right) = x\left(1\right) = 0. \end{cases}$

4. $\int_0^1 \dot{x}^2\, dt \to \text{extr},\, \int_0^1 x\, dt = 1,\, \int_0^1 tx\, dt = 0,\, x\left(0\right) = x\left(1\right) = 0.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 16.

1. $\int_0^1 u^2\, dt \to \text{extr},\, \ddot{x} - x = u,\, x\left(0\right) = \dot{x}\left(0\right) = 0,\, x\left(1\right) = \text{sh}\left(1\right), \dot{x}\left(1\right) = \text{ch}\left(1\right) + \text{sh}\left(1\right).$

2. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x^2\right)\, dt \to \text{extr}, \\ \int_0^1 xe^{-t}\, dt = \left(1 - 3e^{-2}\right)/4, \\ x\left(0\right) = 0,\, x\left(1\right) = 1/e. \end{cases}$

3. $\begin{cases} \int_0^{\pi/2} u^2\, dt \to \text{extr}, \\ \ddot{x} + x = u, \\ x\left(\frac{\pi}{2}\right) = \dot{x}\left(0\right) = 0, \\ x\left(0\right) = 1,\, \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

4. $\begin{cases} \int_0^\pi \left(\ddot{x}^2 - x^2\right)\, dt \to \text{extr}, \\ x\left(0\right) = 0,\, x\left(\pi\right) = \text{sh}\,\pi, \\ \dot{x}\left(0\right) = 0,\, \dot{x}\left(\pi\right) = \text{ch}\,\pi. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 17.

1. $\begin{cases} \int_1^e (t+1)\, t\ddot{x}^2\, dt \to \text{extr}, \\ x(1) = 0,\ x(e) = e, \\ \dot{x}(1) = 1,\ \dot{x}(e) = 2. \end{cases}$

2. $\int_1^e \left(2x - t^2\dot{x}^2\right)\, dt \to \text{extr},\ x(1) = e,\ x(e) = 0.$

3. $\begin{cases} \int_0^T \dot{x}^3\, dt \to \text{extr}, \\ x(0) = 0,\ T + x(T) = 1. \end{cases}$

4. $\begin{cases} \int_0^{t_1} \left(\dot{x}^2 + x + 2\right)\, dt \to \text{extr}, \\ x(0) = 0. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 18.

1. $\int_0^{t_1} \left(\dot{x}^2 + x\right)\, dt \to \text{extr},\ x(t_1) = t_1.$

2. $\begin{cases} \int_0^{\pi} \left(\ddot{x}^2 - \dot{x}^2\right)\, dt \to \text{extr}, \\ x(0) = 0,\ x(\pi) = 1, \\ \dot{x}(0) = 0,\ \dot{x}(\pi) = 0. \end{cases}$

3. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x^2 + 4x \operatorname{sh} t\right)\, dt \to \text{extr}, \\ x(0) = 0. \end{cases}$

4. $\begin{cases} \int_0^{\pi} \dot{x}^2\, dt \to \text{extr}, \\ \int_0^1 x^2\, dt = 1, \\ x(0) = x(1) = 0. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 19.

1. $\begin{cases} \int_0^1 \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 tx\,dt = 0, \\ x(0) = -4,\ x(1) = 4. \end{cases}$

2. $\begin{cases} \int_0^\pi \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 x^2\,dt = 1, \\ x(0) = x(1) = 0. \end{cases}$

3. $\int_1^e \left(2x - t^2\dot{x}^2\right)\,dt \to \text{extr},\ x(1) = e,\ x(e) = 0.$

4. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x\right)\,dt \to \text{extr}, \\ x(1) = 0. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 20.

1. $\int_0^{\pi/2} u^2\,dt \to \text{extr},\ \ddot{x} + x = u,\ x\left(\frac{\pi}{2}\right) = \dot{x}(0) = 0,\ x(0) = 1, \dot{x}\left(\frac{\pi}{2}\right) = 1.$

2. $\begin{cases} \int_0^{t_1} \left(\dot{x}^2 + x + 2\right)\,dt \to \text{extr}, \\ x(0) = 0. \end{cases}$

3. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x^2\right)\,dt \to \text{extr}, \\ \int_0^1 xe^{-t}\,dt = \left(1 - 3e^{-2}\right)/4, \\ x(0) = 0,\ x(1) = 1/e. \end{cases}$

4. $\int\limits_0^1 x^2\dot{x}^2\,dt \to \text{extr},\ x(0) = 1,\ x(1) = \sqrt{2}.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 21.

1. $\begin{cases} \int_0^1 \left(\dot{x}^2 + \ddot{x}^2\right)\, dt \to \text{extr}, \\ x\,(0) = 0,\ x\,(1) = \text{ch}\,1, \\ \dot{x}\,(0) = 0,\ \dot{x}\,(1) = \text{sh}\,1. \end{cases}$

2. $\begin{cases} \int_1^e (t+1)\, t\ddot{x}^2\, dt \to \text{extr}, \\ x\,(1) = 0,\ x\,(e) = e, \\ \dot{x}\,(1) = 1,\ \dot{x}\,(e) = 2. \end{cases}$

3. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x\right)\, dt \to \text{extr}, \\ x\,(1) = 0. \end{cases}$

4. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x\right)\, dt \to \text{extr}, \\ x\,(0) = 0. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 22.

1. $\int_0^{\pi/2} u^2\, dt \to \text{extr},\ \ddot{x} + x = u,\ x\left(\frac{\pi}{2}\right) = \dot{x}\,(0) = 0,\ x\,(0) = 1, \dot{x}\left(\frac{\pi}{2}\right) = 1.$

2. $\int_0^1 \left(\dot{x}^2 + x\right)\, dt \to \text{extr},\ x\,(0) = 1.$

3. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x^2 + 4x\,\text{sh}\,t\right)\, dt \to \text{extr}, \\ x\,(0) = 0. \end{cases}$

4. $\int\limits_0^1 \dot{x}^2\, dt \to \text{extr},\ \int\limits_0^1 x\, dt = 0,\ x\,(0) = 0,\ x\,(1) = 1.$

Билет № 23.

1. $\int\limits_0^1 x^2\dot{x}^2\,dt \to \text{extr},\ x(0)=1,\ x(1)=\sqrt{2}.$

2. $\begin{cases} \int_0^\pi \left(\ddot{x}^2-\dot{x}^2\right)\,dt \to \text{extr}, \\ x(0)=0,\ x(\pi)=1, \\ \dot{x}(0)=0,\ \dot{x}(\pi)=0. \end{cases}$

3. $\begin{cases} \int_0^1 \left(x^2+u^2\right)\,dt \to \text{extr}, \\ \ddot{x}-x=u, \\ x(1)=\dot{x}(0)=0, \\ x(0)=1, \dot{x}(1)=0. \end{cases}$

4. $\begin{cases} \int_0^1 \left(\dot{x}^2+x^2\right)\,dt \to \text{extr}, \\ \int_0^1 xe^t\,dt=\frac{e^2+1}{4}, \\ x(0)=0,\ x(1)=e. \end{cases}$

Методы математической физики
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Билет № 24.

1. $\begin{cases} \int_0^{\pi/2} u^2\,dt \to \text{extr}, \\ \ddot{x}+x=u, \\ x\left(\frac{\pi}{2}\right)=\dot{x}(0)=0, \\ x(0)=1,\ \dot{x}\left(\frac{\pi}{2}\right)=1. \end{cases}$

2. $\begin{cases} \int_1^e (t+1)\,t\ddot{x}^2\,dt \to \text{extr}, \\ x(1)=0,\ x(e)=e, \\ \dot{x}(1)=1,\ \dot{x}(e)=2. \end{cases}$

3. $\int_0^{\pi/2} u^2\,dt \to \text{extr},\ \ddot{x}+x=u,\ x\left(\frac{\pi}{2}\right)=\dot{x}(0)=0,\ x(0)=1, \dot{x}\left(\frac{\pi}{2}\right)=1.$

4. $\int_1^e \left(2x-t^2\dot{x}^2\right)\,dt \to \text{extr},\ x(1)=e,\ x(e)=0.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 25.

1. $\begin{cases} \int_0^{\pi/4} \left(\dot{x}^2 - x^2\right)\, dt \to \text{extr}, \\ x\,(0) = 1. \end{cases}$

2. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x^2\right)\, dt \to \text{extr}, \\ \int_0^1 xe^t\, dt = \frac{e^2+1}{4}, \\ x\,(0) = 0,\ x\,(1) = e. \end{cases}$

3. $\begin{cases} \int_0^1 \dot{x}^2\, dt \to \text{extr}, \\ \int_0^1 xe^{-t}\, dt = e, \\ x\,(0) = 2e + 1,\ x\,(1) = 2. \end{cases}$

4. $\int_0^{\pi} \dot{x}^2\, dt \to \text{extr},\ \int_0^{\pi} x\cos(t)\ dt = \frac{\pi}{2},\ \int_0^{\pi} x\sin(t)\, dt = \pi + 2,\ x\,(0) = 2,\ x\,(\pi) = 0.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 26.

1. $\begin{cases} \int_1^2 t^3\dot{x}^2\, dt \to \text{extr}, \\ \int_1^2 x\, dt = 2, \\ x\,(1) = 4,\ x\,(2) = 1. \end{cases}$

2. $\begin{cases} \int_1^e \left(2x - t^2\dot{x}^2\right)\, dt \to \text{extr}, \\ x\,(1) = e,\ x\,(e) = 0. \end{cases}$

3. $\int_0^{\pi} \dot{x}^2\, dt \to \text{extr},\ \int_0^{\pi} x\sin(t)\ dt = 0,\ x\,(0) = 0,\ x\,(\pi) = 1.$

4. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x\right)\, dt \to \text{extr}, \\ x\,(1) = 0. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 27.

1. $\int\limits_0^1 \ddot{x}^2\,dt \to \text{extr},\ x(0) = \dot{x}(0) = x(1) = 0,\ \dot{x}(0) = 1.$

2. $\begin{cases} \int_0^1 \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 xe^{-t}\,dt = e, \\ x(0) = 2e+1,\ x(1) = 2. \end{cases}$

3. $\int_0^1 \dot{x}^2\,dt \to \text{extr},\ \int_0^1 x\,e^{-t}\,dt = e,\ x(0) = 2e+1,\ x(1) = 2.$

4. $\int\limits_0^2 \left(\dot{x}^2 - x\right)\,dt \to \text{extr},\ x(0) = 0.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 28.

1. $\begin{cases} \int_0^{\pi/2} u^2\,dt \to \text{extr}, \\ \ddot{x} + x = u, \\ x\left(\frac{\pi}{2}\right) = \dot{x}(0) = 0, \\ x(0) = 1,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

2. $\int_0^1 \dot{x}^2\,dt \to \text{extr},\ \int_0^1 tx\,dt = 0,\ x(0) = -4,\ x(1) = 4.$

3. $\int_0^1 u^2\,dt \to \text{extr},\ \ddot{x} - x = u,\ x(0) = \dot{x}(0) = 0,\ x(1) = \text{sh}(1),\ \dot{x}(1) = \text{ch}(1) + \text{sh}(1).$

4. $\begin{cases} \int_0^1 \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 tx\,dt = 0, \\ x(0) = -4,\ x(1) = 4. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 29.

1. $\int\limits_0^1 x^2\dot{x}^2\,dt \to \text{extr},\ x(0)=1,\ x(1)=\sqrt{2}.$

2. $\begin{cases} \int_0^1 \left(\dot{x}^2+x\right)\,dt \to \text{extr}, \\ x(1)=0. \end{cases}$

3. $\int\limits_0^T \sqrt{1+\dot{x}^2}dt \to \text{extr}, x(0)=0,\ T^2x(T)=1.$

4. $\begin{cases} \int_1^3 \left(t^2\dot{x}^2+t\right)\,dt \to \text{extr}, \\ x(1)=3,\ x(3)=0. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 30.

1. $\begin{cases} \int_0^1 \left(\dot{x}^2+x^2+4x\,\text{sh}\,t\right)\,dt \to \text{extr}, \\ x(0)=0. \end{cases}$

2. $\begin{cases} \int_0^1 \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 tx\,dt=0, \\ x(0)=-4,\ x(1)=4. \end{cases}$

3. $\int\limits_0^T \left(\dot{x}^2-x+1\right)\,dt \to \text{extr},\ x(0)=0.$

4. $\begin{cases} \int_0^1 \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 tx\,dt=0, \\ x(0)=-4,\ x(1)=4. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 31.

1. $\int_0^{\pi} \dot{x}^2\,dt \to \mathrm{extr}$, $\int_0^{\pi} x\sin(t)\,dt = 0$, $x(0) = 0$, $x(\pi) = 1$.

2. $\begin{cases} \int_0^{t_1} \left(\dot{x}^2 + x\right)\,dt \to \mathrm{extr}, \\ x(0) = 1\,. \end{cases}$

3. $\begin{cases} \int_0^{1} \left(\dot{x}^2 + x\right)\,dt \to \mathrm{extr}, \\ x(0) = 0. \end{cases}$

4. $\begin{cases} \int_0^{1} \left(x^2 + u^2\right)\,dt \to \mathrm{extr}, \\ \ddot{x} - x = u, \\ x(1) = \dot{x}(0) = 0, \\ x(0) = 1, \dot{x}(1) = 0. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 32.

1. $\begin{cases} \int_0^{1} \ddot{x}^2\,dt \to \mathrm{extr}, \\ x(0) = \dot{x}(0) = 0, \\ x(1) = 0,\ \dot{x}(1) = 1. \end{cases}$

2. $\int\limits_0^{\pi/2} \left(\dot{x}^2 - x^2 - 2x\right)\,dt - 2x^2(0) - x^2\left(\frac{\pi}{2}\right) \to \mathrm{extr}\,.$

3. $\begin{cases} \int_0^{\pi/2} u^2\,dt \to \mathrm{extr}, \\ \ddot{x} + x = u, \\ x\left(\frac{\pi}{2}\right) = \dot{x}(0) = 0, \\ x(0) = 1,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

4. $\begin{cases} \int_0^{1} \left(\dot{x}^2 + x^2 + 4x\,\mathrm{sh}\,t\right)\,dt \to \mathrm{extr}, \\ x(0) = 0. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 33.

1. $\int_0^{\pi/2} u^2\,dt \to \text{extr},\ \ddot{x} + x = u,\ x\left(\frac{\pi}{2}\right) = \dot{x}\,(0) = 0,\ x\,(0) = 1, \dot{x}\left(\frac{\pi}{2}\right) = 1.$

2. $\begin{cases} \int_0^1 \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 xe^{-t}\,dt = e, \\ x\,(0) = 2e + 1,\ x\,(1) = 2. \end{cases}$

3. $\int\limits_0^{\pi/2} \left(\dot{x}^2 - x^2 - 2x\right)\,dt - 2x^2\,(0) - x^2\left(\frac{\pi}{2}\right) \to \text{extr}\,.$

4. $\int_0^1 \dot{x}^2\,dt \to \text{extr},\ \int_0^1 x\,dt = -\frac{3}{2},\ \int_0^1 tx\,dt = -2,\ x\,(0) = 2,\ x\,(1) = -14.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 34.

1. $\begin{cases} \int_0^{\pi} \left(\ddot{x}^2 - \dot{x}^2\right)\,dt \to \text{extr}, \\ x\,(0) = 0,\ x\,(\pi) = 1, \\ \dot{x}\,(0) = 0,\ \dot{x}\,(\pi) = 0. \end{cases}$

2. $\int_0^1 u^2\,dt \to \text{extr},\ \dot{x} - 2x = u,\ x\,(0) = x\,(1) = 0.$

3. $\int\limits_0^1 \ddot{x}_2^2\,dt \to \text{extr},\ \dot{x}_1 = x_2,\ x_1\,(0) = x_2\,(0) = 0,\ x_1\,(1)\,.$

4. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x^2 + 4x\,\text{sh}\,t\right)\,dt \to \text{extr}, \\ x\,(0) = 0. \end{cases}$

Билет № 35.

1. $\int\limits_0^T \sqrt{1+\dot{x}^2}dt \to \text{extr}, x\left(0\right)=0,\, T^2x\left(T\right)=1.$

2. $\begin{cases} \int_0^\pi \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 x^2\,dt = 1, \\ x\left(0\right) = x\left(1\right) = 0. \end{cases}$

3. $\begin{cases} \int_0^1 \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 tx\,dt = 0, \\ x\left(0\right) = -4,\, x\left(1\right) = 4. \end{cases}$

4. $\int\limits_0^1 x^2\dot{x}^2\,dt \to \text{extr},\, x\left(0\right)=1,\, x\left(1\right)=\sqrt{2}.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 36.

1. $\int\limits_0^1 \dot{x}^2\,dt \to \text{extr},\, \int\limits_0^1 x\,dt = 0,\, x\left(0\right)=0,\, x\left(1\right)=1.$

2. $\begin{cases} \int_1^2 t^2\dot{x}^2\,dt \to \text{extr}, \\ x\left(1\right)=3,\, x\left(2\right)=1. \end{cases}$

3. $\begin{cases} \int_0^1 \left(\dot{x}_1\dot{x}_2 + 6x_1t + 12x_2t^2\right)\,dt \to \text{extr}, \\ x_1\left(0\right) = x_2\left(0\right) = 0, \\ x_1\left(\frac{\pi}{2}\right) = 1,\, x_2\left(\frac{\pi}{2}\right) = -1. \end{cases}$

4. $\int\limits_0^2 \left(\dot{x}^2 - x\right)\,dt \to \text{extr},\, x\left(0\right)=0.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 37.

1. $\int_0^1 u^2\,dt \to \text{extr},\ \dot{x} - x = 3u,\ x(0) = 0,\ x(1) = 1.$

2. $\begin{cases} \int_0^1 u^2\,dt \to \text{extr}, \\ \ddot{x} - x = u, \\ x(0) = \dot{x}(0) = 0, \\ x(1) = \operatorname{sh} 1, \\ \dot{x}(1) = \operatorname{sh} 1 + \operatorname{ch} 1. \end{cases}$

3. $\int_0^1 u^2\,dt \to \text{extr},\ \dot{x} - 2x = u,\ x(0) = x(1) = 0.$

4. $\int_0^1 \left(\dot{x}^2 + x\right)\,dt \to \text{extr},\ x(0) = 1.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 38.

1. $\int_0^1 \dot{x}^2\,dt \to \text{extr},\ \int_0^1 x\,dt = 1,\ \int_0^1 tx\,dt = 0,\ x(0) = x(1) = 0.$

2. $\begin{cases} \int_0^{\pi/4} \left(\dot{x}^2 - x^2\right)\,dt \to \text{extr}, \\ x(0) = 1. \end{cases}$

3. $\begin{cases} \int_0^1 \left(x^2 + u^2\right)\,dt \to \text{extr}, \\ \ddot{x} - 4x = u, \\ x(1) = \dot{x}(0) = 0, \\ x(0) = 0,\ \dot{x}(1) = 1. \end{cases}$

4. $\int_0^{\pi} \dot{x}^2\,dt \to \text{extr},\ \int_0^{\pi} x\cos(t)\,dt = \frac{\pi}{2},\ \int_0^{\pi} x\sin(t)\,dt = \pi + 2,\ x(0) = 2,\ x(\pi) = 0.$

Билет № 39.

1. $\int\limits_0^1 e^x \dot{x}^2\,dt + 4e^{x(0)} + 32e^{-x(1)} \to \text{extr}\,.$

2. $\begin{cases} \int_1^e (t+1)\,t\ddot{x}^2\,dt \to \text{extr}, \\ x(1) = 0,\ x(e) = e, \\ \dot{x}(1) = 1,\ \dot{x}(e) = 2. \end{cases}$

3. $\int_0^1 \dot{x}^2\,dt \to \text{extr},\ \int_0^1 x\,dt = 1,\ \int_0^1 tx\,dt = 0,\ x(0) = x(1) = 0.$

4. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x\right)\,dt \to \text{extr}, \\ x(0) = 0. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 40.

1. $\begin{cases} \int_0^1 \left(x^2 + u^2\right)\,dt \to \text{extr}, \\ \ddot{x} - x = u, \\ x(1) = \dot{x}(0) = 0, \\ x(0) = 1, \dot{x}(1) = 0. \end{cases}$

2. $\begin{cases} \int_1^e \left(2x - t^2\dot{x}^2\right)\,dt \to \text{extr}, \\ x(1) = e,\ x(e) = 0. \end{cases}$

3. $\begin{cases} \int_0^{\pi/2} \left(x^2 + u^2\right)\,dt \to \text{extr}, \\ \ddot{x} + x = u, \\ x\left(\frac{\pi}{2}\right) = \dot{x}(0) = 1, \\ x(0) = 0,\ \dot{x}\left(\frac{\pi}{2}\right) = 0. \end{cases}$

4. $\begin{cases} \int_0^1 u^2\,dt \to \text{extr}, \\ \ddot{x} - x = 3u, \\ x(0) = 0,\ x(1) = 1, \\ \dot{x}(0) = 0,\ \dot{x}(1) = 1. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 41.

1. $\int_0^1 u^2\,dt \to \text{extr},\ \dot{x}+x=2u,\ x(0)=1,\ x(1)=0.$

2. $\int\limits_0^T \sqrt{1+\dot{x}^2}dt \to \text{extr}, x(0)=0,\ T^2x(T)=1.$

3. $\begin{cases} \int_0^{\pi/2} u^2\,dt \to \text{extr}, \\ \ddot{x}+x=u, \\ x\left(\frac{\pi}{2}\right)=\dot{x}(0)=0, \\ x(0)=1,\ \dot{x}\left(\frac{\pi}{2}\right)=1. \end{cases}$

4. $\int_0^1 \left(\dot{x}^2+x\right)\,dt \to \text{extr},\ x(1)=0.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 42.

1. $\begin{cases} \int_1^2 t^3\dot{x}^2\,dt \to \text{extr}, \\ \int_1^2 x\,dt=2, \\ x(1)=4,\ x(2)=1. \end{cases}$

2. $\begin{cases} \int_0^{\pi/2} u^2\,dt \to \text{extr}, \\ \ddot{x}+x=u, \\ x\left(\frac{\pi}{2}\right)=\dot{x}(0)=0, \\ x(0)=1,\ \dot{x}\left(\frac{\pi}{2}\right)=1. \end{cases}$

3. $\int_1^2 t^2\dot{x}^2\,dt \to \text{extr},\ x(1)=3,\ x(2)=1.$

4. $\begin{cases} \int_0^{t_1} \left(\dot{x}^2+x+2\right)\,dt \to \text{extr}, \\ x(0)=0. \end{cases}$

Билет № 43.

1. $\left\{ \begin{array}{l} \int_0^1 \left(\dot{x}^2 + x^2 + 4x \operatorname{sh} t\right)\, dt \to \text{extr}, \\ x\,(0) = 0. \end{array} \right.$

2. $\int_0^1 \dot{x}^2\, dt \to \text{extr}$, $\int_0^1 x\, e^{-t}\, dt = e$, $x\,(0) = 2e + 1$, $x\,(1) = 2$.

3. $\left\{ \begin{array}{l} \int_0^\pi \dot{x}^2\, dt \to \text{extr}, \\ \int_0^1 x^2\, dt = 1, \\ x\,(0) = x\,(1) = 0. \end{array} \right.$

4. $\int_0^1 \dot{x}^2\, dt \to \text{extr}$, $\int_0^1 x\, e^{-t}\, dt = e$, $x\,(0) = 2e + 1$, $x\,(1) = 2$.

Методы математической физики
Контрольная по вариационному исчислению

Билет № 44.

1. $\left\{ \begin{array}{l} \int_0^1 \dot{x}^2\, dt \to \text{extr}, \\ \int_0^1 x e^{-t}\, dt = e, \\ x\,(0) = 2e + 1,\ x\,(1) = 2. \end{array} \right.$

2. $\int_0^1 \dot{x}^2\, dt \to \text{extr}$, $\int_0^1 x\, dt = 1$, $\int_0^1 tx\, dt = 0$, $x\,(0) = x\,(1) = 0$.

3. $\left\{ \begin{array}{l} \int_1^2 t^2 \ddot{x}^2\, dt \to \text{extr}, \\ \int_1^2 tx\, dt = \frac{7}{3}, \\ x\,(1) = 1,\ x\,(2) = 2. \end{array} \right.$

4. $\left\{ \begin{array}{l} \int_0^{t_1} \left(\dot{x}^2 + x\right)\, dt \to \text{extr}, \\ x\,(0) = 1\,. \end{array} \right.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 45.

1. $\begin{cases} \int_0^\pi \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 x^2\,dt = 1, \\ x(0) = x(1) = 0. \end{cases}$

2. $\begin{cases} \int_1^e (t+1)\,t\ddot{x}^2\,dt \to \text{extr}, \\ x(1) = 0,\ x(e) = e, \\ \dot{x}(1) = 1,\ \dot{x}(e) = 2. \end{cases}$

3. $\int\limits_0^T \left(\dot{x}^2 - x + 1\right)\,dt \to \text{extr},\ x(0) = 0.$

4. $\int_0^1 \dot{x}^2\,dt \to \text{extr},\ \int_0^1 x\,dt = -\frac{3}{2},\ \int_0^1 tx\,dt = -2,\ x(0) = 2,\ x(1) = -14.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 46.

1. $\begin{cases} \int_1^e (t+1)\,t\ddot{x}^2\,dt \to \text{extr}, \\ x(1) = 0,\ x(e) = e, \\ \dot{x}(1) = 1,\ \dot{x}(e) = 2. \end{cases}$

2. $\begin{cases} \int_0^1 \dot{x}^2\,dt \to \text{extr}, \\ \int_0^1 xe^{-t}\,dt = e, \\ x(0) = 2e+1,\ x(1) = 2. \end{cases}$

3. $\begin{cases} \int_0^1 \left(x^2 + u^2\right)\,dt \to \text{extr}, \\ \ddot{x} - x = u, \\ x(1) = \dot{x}(0) = 0, \\ x(0) = 1, \dot{x}(1) = 0. \end{cases}$

4. $\begin{cases} \int_0^1 u^2\,dt \to \text{extr}, \\ \ddot{x} - x = u, \\ x(0) = \dot{x}(0) = 0, \\ x(1) = \operatorname{sh} 1, \\ \dot{x}(1) = \operatorname{sh} 1 + \operatorname{ch} 1. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 47.

1. $\begin{cases} \int_0^{\pi/4} \left(\dot{x}^2 - x^2\right)\, dt \to \text{extr}, \\ x\,(0) = 1. \end{cases}$

2. $\int_0^1 \dot{x}^2\, dt \to \text{extr},\ \int_0^1 x\, e^{-t}\, dt = e,\ x\,(0) = 2e + 1,\ x\,(1) = 2.$

3. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x\right)\, dt \to \text{extr}, \\ x\,(0) = 0. \end{cases}$

4. $\int\limits_0^{\pi/2} \left(\dot{x}^2 - x^2 - 2x\right)\, dt - 2x^2\,(0) - x^2\left(\frac{\pi}{2}\right) \to \text{extr}\,.$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 48.

1. $\int\limits_0^1 \left(\dot{x}^2 - 4\dot{x}^3 x + 2t\dot{x}^4\right)\, dt \to \text{extr}, x\,(0) = 0,\ x\,(1) = 0.$

2. $\int_1^e \left(t\dot{x}^2 + 2x\right)\, dt \to \text{extr},\ x\,(1) = 1,\ x\,(e) = 0.$

3. $\begin{cases} \int_0^{\pi/2} u^2\, dt \to \text{extr}, \\ \ddot{x} + x = u, \\ x\left(\frac{\pi}{2}\right) = \dot{x}\,(0) = 0, \\ x\,(0) = 1,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

4. $\int_0^1 \dot{x}^2\, dt \to \text{extr},\ \int_0^1 tx\, dt = 0,\ x\,(0) = -4,\ x\,(1) = 4.$

Билет № 49.

1. $\int_0^1 \dot{x}^2\,dt \to \text{extr}$, $\int_0^1 x\,e^{-t}\,dt = e$, $x(0) = 2e+1$, $x(1) = 2$.

2. $\begin{cases} \int_0^1 \left(x^2+u^2\right)\,dt \to \text{extr}, \\ \ddot{x} - 9\dot{x} = u, \\ x(1) = \dot{x}(0) = 0, \\ x(0) = 1,\ \dot{x}(1) = 0. \end{cases}$

3. $\begin{cases} \int_1^e (t+1)\,t\ddot{x}^2\,dt \to \text{extr}, \\ x(1) = 0,\ x(e) = e, \\ \dot{x}(1) = 1,\ \dot{x}(e) = 2. \end{cases}$

4. $\int_0^1 \dot{x}^2\,dt \to \text{extr}$, $\int_0^1 x\,dt = 1$, $\int_0^1 tx\,dt = 0$, $x(0) = x(1) = 0$.

Методы математической физики
Контрольная по вариационному исчислению

Билет № 50.

1. $\begin{cases} \int_0^{\pi/2} u^2\,dt \to \text{extr}, \\ \ddot{x} + x = u, \\ x\left(\frac{\pi}{2}\right) = \dot{x}(0) = 0, \\ x(0) = 1,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

2. $\int\limits_0^1 \ddot{x}^2\,dt \to \text{extr}$, $x(0) = \dot{x}(0) = x(1) = 0$, $\dot{x}(0) = 1$.

3. $\int\limits_0^1 e^x\,\dot{x}^2\,dt + 4e^{x(0)} + 32e^{-x(1)} \to \text{extr}\,.$

4. $\begin{cases} \int_0^{\pi/2} \left(x^2+4u^2\right)\,dt \to \text{extr}, \\ \ddot{x} + 4x = u, \\ x\left(\frac{\pi}{2}\right) = 1,\ \dot{x}(0) = 0, \\ x(0) = 0,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 51.

1. $\begin{cases} \int_0^1 \left(\dot{x}_1\dot{x}_2 + 6x_1 t + 12x_2 t^2\right)\, dt \to \text{extr}, \\ x_1(0) = x_2(0) = 0, \\ x_1\left(\frac{\pi}{2}\right) = 1,\ x_2\left(\frac{\pi}{2}\right) = -1. \end{cases}$

2. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x\right)\, dt \to \text{extr}, \\ x(0) = 0. \end{cases}$

3. $\begin{cases} \int_0^\pi \left(\ddot{x}^2 - x^2\right)\, dt \to \text{extr}, \\ x(0) = 0,\ x(\pi) = \operatorname{sh}\pi, \\ \dot{x}(0) = 0,\ \dot{x}(\pi) = \operatorname{ch}\pi. \end{cases}$

4. $\begin{cases} \int_0^{\pi/2} \left(x^2 + 4u^2\right)\, dt \to \text{extr}, \\ \ddot{x} + 4x = u, \\ x\left(\frac{\pi}{2}\right) = 1,\ \dot{x}(0) = 0, \\ x(0) = 0,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

Методы математической физики
Контрольная по вариационному исчислению

Билет № 52.

1. $\int_0^1 \dot{x}^2\, dt \to \text{extr},\ \int_0^1 x\, dt = 1,\ \int_0^1 tx\, dt = 0,\ x(0) = x(1) = 0.$

2. $\int_0^1 \dot{x}^2\, dt \to \text{extr},\ \int_0^1 x\, dt = -\frac{3}{2},\ \int_0^1 tx\, dt = -2,\ x(0) = 2,\ x(1) = -14.$

3. $\begin{cases} \int_0^{\pi/2} u^2\, dt \to \text{extr}, \\ \ddot{x} + x = u, \\ x\left(\frac{\pi}{2}\right) = \dot{x}(0) = 0, \\ x(0) = 1,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

4. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x^2\right)\, dt \to \text{extr}, \\ \int_0^1 xe^t\, dt = \frac{e^2+1}{4}, \\ x(0) = 0,\ x(1) = e. \end{cases}$

Билет № 53.

1. $\begin{cases} \int_0^1 \left(\dot{x}^2 + x^2 + 4x \operatorname{sh} t\right)\, dt \to \operatorname{extr}, \\ x\,(0) = 0. \end{cases}$

2. $\int_0^{t_1} \left(\dot{x}^2 + x\right)\, dt \to \operatorname{extr},\ x\,(t_1) = t_1.$

3. $\int\limits_0^1 e^x \dot{x}^2\, dt + 4e^{x(0)} + 32e^{-x(1)} \to \operatorname{extr}.$

4. $\int_1^e \left(2x - t^2\dot{x}^2\right)\, dt \to \operatorname{extr},\ x\,(1) = e,\ x\,(e) = 0.$

Билет № 54.

1. $\int\limits_0^1 \ddot{x}_2^2\, dt \to \operatorname{extr},\ \dot{x}_1 = x_2,\ x_1\,(0) = x_2\,(0) = 0,\ x_1\,(1).$

2. $\begin{cases} \int_0^T \left(\dot{x}^2 - x^2\right)\, dt \to \operatorname{extr}, \\ x\,(0) = 0. \end{cases}$

3. $\begin{cases} \int_0^{\pi/2} u^2\, dt \to \operatorname{extr}, \\ \ddot{x} + x = u, \\ x\left(\frac{\pi}{2}\right) = \dot{x}\,(0) = 0, \\ x\,(0) = 1,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

4. $\begin{cases} \int_0^1 u^2\, dt \to \operatorname{extr}, \\ \ddot{x} - x = u, \\ x\,(0) = \dot{x}\,(0) = 0, \\ x\,(1) = \operatorname{sh} 1, \\ \dot{x}\,(1) = \operatorname{sh} 1 + \operatorname{ch} 1. \end{cases}$

Билет № 55.

1. $\int\limits_0^2 \left(\dot{x}^2 - x\right)\, dt \to \text{extr},\ x\,(0) = 0.$

2. $\begin{cases} \int_0^1 \dot{x}^2\, dt \to \text{extr}, \\ \int_0^1 tx\, dt = 0, \\ x\,(0) = -4,\ x\,(1) = 4. \end{cases}$

3. $\begin{cases} \int_0^1 \left(x^2 + u^2\right)\, dt \to \text{extr}, \\ \ddot{x} - 4x = u, \\ x\,(1) = \dot{x}\,(0) = 0, \\ x\,(0) = 0,\ \dot{x}\,(1) = 1. \end{cases}$

4. $\begin{cases} \int_0^1 \left(x^2 + u^2\right)\, dt \to \text{extr}, \\ \ddot{x} - x = u, \\ x\,(1) = \dot{x}\,(0) = 0, \\ x\,(0) = 1,\ \dot{x}\,(1) = 0. \end{cases}$

Билет № 56.

1. $\begin{cases} \int_0^1 u^2\, dt \to \text{extr}, \\ \ddot{x} - x = u, \\ x\,(0) = \dot{x}\,(0) = 0, \\ x\,(1) = \operatorname{sh} 1, \\ \dot{x}\,(1) = \operatorname{sh} 1 + \operatorname{ch} 1. \end{cases}$

2. $\int\limits_0^1 \ddot{x}^2\, dt \to \text{extr},\ x\,(0) = \dot{x}\,(0) = x\,(1) = 0,\ \dot{x}\,(0) = 1.$

3. $\begin{cases} \int_0^{\pi/2} \left(x^2 + 4u^2\right)\, dt \to \text{extr}, \\ \ddot{x} + 4x = u, \\ x\left(\frac{\pi}{2}\right) = 1,\ \dot{x}\,(0) = 0, \\ x\,(0) = 0,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

4. $\int_0^1 \dot{x}^2\, dt \to \text{extr},\ \int_0^1 x\, e^{-t}\, dt = e,\ x\,(0) = 2e + 1,\ x\,(1) = 2.$

Билет № 57.

1. $\begin{cases} \int_0^{\pi/2} \left(x^2 + 4u^2\right)\, dt \to \text{extr}, \\ \ddot{x} + 4x = u, \\ x\left(\frac{\pi}{2}\right) = 1,\ \dot{x}\,(0) = 0, \\ x\,(0) = 0,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

2. $\int_0^{\pi} \dot{x}^2\, dt \to \text{extr},\ \int_0^{\pi} x\cos(t)\ dt = \frac{\pi}{2},\ \int_0^{\pi} x\sin(t)\, dt = \pi + 2,\ x\,(0) = 2,\ x\,(\pi) = 0.$

3. $\begin{cases} \int_0^{t_1} \left(\dot{x}^2 + x\right)\, dt \to \text{extr}, \\ x\,(0) = 1\,. \end{cases}$

4. $\begin{cases} \int_0^{\pi/2} \left(x^2 + 4u^2\right)\, dt \to \text{extr}, \\ \ddot{x} + 4x = u, \\ x\left(\frac{\pi}{2}\right) = 1,\ \dot{x}\,(0) = 0, \\ x\,(0) = 0,\ \dot{x}\left(\frac{\pi}{2}\right) = 1. \end{cases}$

Билет № 58.

1. $\begin{cases} \int_0^{\pi} \left(\ddot{x}^2 - x^2\right)\, dt \to \text{extr}, \\ x\,(0) = 0,\ x\,(\pi) = \operatorname{sh}\pi, \\ \dot{x}\,(0) = 0,\ \dot{x}\,(\pi) = \operatorname{ch}\pi. \end{cases}$

2. $\begin{cases} \int_0^{1} \left(\dot{x}_1\dot{x}_2 + 6x_1 t + 12x_2 t^2\right)\, dt \to \text{extr}, \\ x_1\,(0) = x_2\,(0) = 0, \\ x_1\left(\frac{\pi}{2}\right) = 1,\ x_2\left(\frac{\pi}{2}\right) = -1. \end{cases}$

3. $\begin{cases} \int_1^{2} t^2\ddot{x}^2\, dt \to \text{extr}, \\ \int_1^{2} tx\, dt = \frac{7}{3}, \\ x\,(1) = 1,\ x\,(2) = 2. \end{cases}$

4. $\begin{cases} \int_0^{1} \dot{x}^2\, dt \to \text{extr}, \\ \int_0^{1} tx\, dt = 0, \\ x\,(0) = -4,\ x\,(1) = 4. \end{cases}$

Билет № 59.

1. $\int\limits_0^1 x^2\dot{x}^2\,dt \to \mathrm{extr},\ x(0)=1,\ x(1)=\sqrt{2}.$

2. $\begin{cases} \int_0^1 \left(\dot{x}^2+x\right)\,dt \to \mathrm{extr}, \\ x(0)=0. \end{cases}$

3. $\begin{cases} \int_0^1 \left(\dot{x}^2+x^2\right)\,dt \to \mathrm{extr}, \\ \int_0^1 xe^t\,dt = \frac{e^2+1}{4}, \\ x(0)=0,\ x(1)=e. \end{cases}$

4. $\begin{cases} \int_0^1 \left(\dot{x}^2+\ddot{x}^2\right)\,dt \to \mathrm{extr}, \\ x(0)=0,\ x(1)=\mathrm{ch}\,1, \\ \dot{x}(0)=0,\ \dot{x}(1)=\mathrm{sh}\,1. \end{cases}$

Билет № 60.

1. $\begin{cases} \int_0^{t_1} \left(\dot{x}^2+x\right)\,dt \to \mathrm{extr}, \\ x(t_1)=t_1. \end{cases}$

2. $\begin{cases} \int_0^{\pi/2} u^2\,dt \to \mathrm{extr}, \\ \ddot{x}+x=u, \\ x\left(\frac{\pi}{2}\right)=\dot{x}(0)=0, \\ x(0)=1,\ \dot{x}\left(\frac{\pi}{2}\right)=1. \end{cases}$

3. $\begin{cases} \int_0^1 u^2\,dt \to \mathrm{extr}, \\ \ddot{x}-x=u, \\ x(0)=\dot{x}(0)=0, \\ x(1)=\mathrm{sh}\,1, \\ \dot{x}(1)=\mathrm{sh}\,1+\mathrm{ch}\,1. \end{cases}$

4. $\int_0^1 \dot{x}^2\,dt \to \mathrm{extr},\ \int_0^1 x\,e^{-t}\,dt = e,\ x(0)=2e+1,\ x(1)=2.$